2. ***Toothpaste Factory Problem*** A "rule of thumb" used by chemical engineers to estimate the cost of a chemical factory is that the cost is directly proportional to the 0.6 power of the amount of chemical the factory produces per unit time. Suppose that a toothpaste factory which turns out 15 tons per day of toothpaste costs 43 million dollars to build.

a. Write the particular equation expressing the cost of a toothpaste factory in terms of the number of tons per day it produces.

b. Predict the cost of building a factory to produce
   i. 500 tons per day,
   ii. 0.07 tons per day.

c. If you double the manufacturing capacity of a toothpaste factory will the cost be double, more than double, or less than double? Justify your answer.

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a) $y = Kx^n$

$C = Ka^n$

$43 = K\,15^{.6}$

$K = 8.469$

$C = 8.469a^{.6}$

$\boxed{C = 8.469a^{.6}}$

b) i) $C = 8.469(500^{.6})$

$C = \$352,529,693$

ii) $C = 8.469(.07^{.6})$

$C = \$1,717,477$

or

i) \$352.5 million

ii) \$1.717 million

---

c) $C_{old} = 8.469(a^{.6})$ ← before doubling capacity

$C_{new} = 12.8366a^{.6}$

$\dfrac{12.8366}{8.469} \approx 1.5$

Less than double

OR $C_{new} = 8.469(2^{.6})a^{.6}$

$= 2^{.6}(8.469a^{.6})$

$= 1.516(C_{old})$

less than double

(8.5) Variation Functions Day 1 thr p437

(5) Let d = diameter
m = height in meters

a) $d = Km^{\frac{3}{2}}$

$14.5 = K5^{\frac{3}{2}}$

$K \approx 1.29$

$\boxed{d = 1.3m^{\frac{3}{2}}}$

b) $d = 1.3(50)^{\frac{3}{2}}$

$d \approx 459.619$

$d = 460$ meters

$\frac{460}{14.5} \approx 31.7$

32 times as big

c) $985 = 1.3m^{\frac{3}{2}}$

$m \approx 83.11$

83 meters

(7) a) $p = K\ell^{\frac{1}{2}}$

$.55 = K(.3)^{\frac{1}{2}}$

$K \approx 1.00415$

$p = 1.00415\ell^{\frac{1}{2}}$

b) $1 = 1.00415\ell^{\frac{1}{2}}$

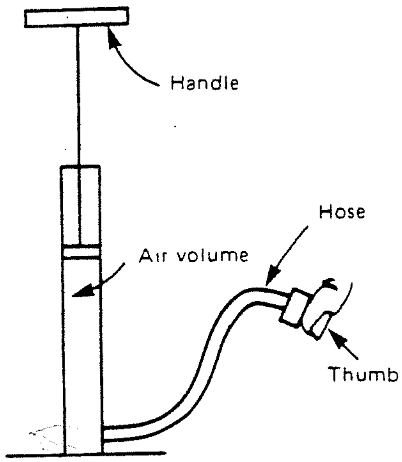
$\left(\frac{1}{1.00415}\right)^2 = \left(\ell^{\frac{1}{2}}\right)^2$

$\ell \approx .99175\,m$

c) $10 = 1.00415\ell^{\frac{1}{2}}$

$\ell \approx 99.175$

6. ***Tire Pump Problem*** If you compress a gas quickly, such as by pushing down the handle of the tire pump on the sketch, the heat generated by compression does not have time to escape, and the gas warms up. In this case, the pressure varies inversely with the 1.4 power of the volume. (In problem 7 of Section 7-11, Boyle's Law said that the pressure varied inversely with the *first* power of the volume, but that was because the temperature remained *constant*.)



a. Suppose that when the pump handle is fully extended, the volume is 40 cubic inches, and the pressure is 15 pounds per square inch (psi), the normal atmospheric pressure. Write the particular equation expressing pressure in terms of volume.

b. You hold your thumb over the end of the hose, then push down on the handle, reducing the volume to 20 cubic inches. Predict the pressure.

c. Suppose that you connect the hose to a tire containing air at 50 psi. When you push down the handle, the pressure in the pump will increase until it just equals the pressure in the tire. What will the volume of air in the pump be when its pressure just reaches 50 psi?

d. Draw the graph of pressure versus volume in the domain $0 < \text{volume} \le 40$ cubic inches with the pump connected to the tire. Take into account the fact that when the pressure reaches 50 psi, the air flows into the tire rather than being compressed to a higher pressure.

a) $p = \frac{k}{V^{1.4}}$

$15 = \frac{k}{40^{1.4}}$

$K = 2624.06897$

$K = 2624.069$

$p = \frac{2624.069}{V^{1.4}}$

or

$p = \frac{2624}{V^{1.4}}$ (Dani's)

b) $p = \frac{2624}{V^{1.4}}$

$p = \frac{2624}{20^{1.4}}$

$p = 39.585$

$p \approx 39.59$ psi

c) $50 = \frac{2624}{V^{1.4}}$

$V^{1.4} = \frac{2624}{50}$

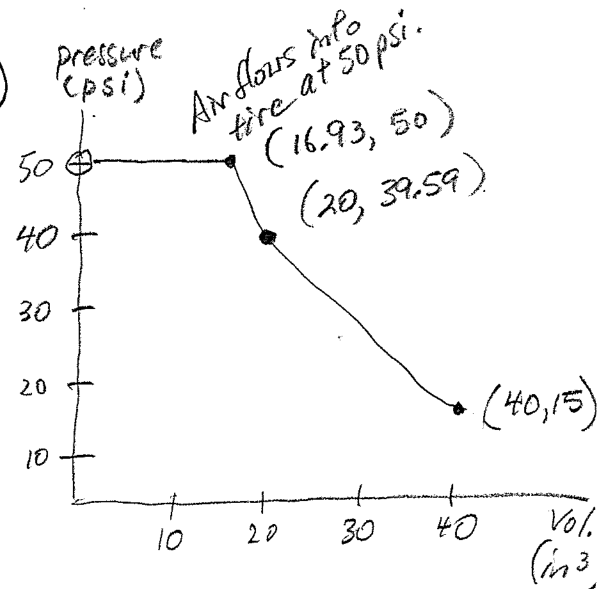
$1.4 \log V = \log(52.48)$

$\log V = 1.2285$

$V \approx 16.9265$

$\boxed{V \approx 16.93 \text{ cu. in.}}$

d)



Variation Functions (8-5) Day 1

Microwave oven problem.

9 a) Not a direct variation b/c double # slices but time does not double.

b) $(2, 1.75)$ $\times 2$ $(4, 2.5)$ $\frac{10}{7}$

$T = KS^n$ ← general EQ

To figure out $2^? = \frac{10}{7} \Longrightarrow 2^x = \frac{10}{7}$

$\log 2^x = \log \frac{10}{7}$

$x \log 2 = \log 10 - \log 7$

$x = 0.514573$

$\hookrightarrow 0.5146$

$T = KS^{.5146}$

$2 = K\,1.75^{.5146}$

$K = 1.22500$

$\hookrightarrow 1.225$

So, particular EQ: $\boxed{T = 1.225\,S^{.5146}}$

c) $S = 8$ slices $\quad T = 3.57$ minutes

$S = 6$ slices $\quad T = 3.08$ min.

$S = 1$ slice $\quad T = 1.225$ min.

d) $30 = 1.225\,S^{.5146}$

$\frac{30}{1.225} = S^{.5146}$

solve 2 ways:

$\left(\frac{30}{1.225}\right)^{\frac{1}{.5146}} = S^{0.5146\left(\frac{1}{.5146}\right)}$

$S \approx 500.2$

or

$\log\left(\frac{30}{1.225}\right) = \log S^{.5146}$

$\log S = \frac{\log\left(\frac{30}{1.225}\right)}{.5146}$

$S \approx 500.2$

$\boxed{500 \text{ slices}}$

e) domain:

$0 \le x \le 500$

range:

$0 \le y \le 30$

f) If your oven cannot hold 500 slices of bacon, then a smaller domain.

variation functions p441/11

(11) a)

days	sq. ft
3	120
6	156
9	203
12	264

exponential function

$y = a \cdot b^x$

b) $120 = ab^3$

$264 = ab^{12}$

$\frac{264}{120} = b^9$

$b \approx 1.09155...$

$120 = a(1.09155)^3$

$a \approx 92.2657...$

$\boxed{y = (92.2657)(1.09155)^x}$

c)

day	square feet
6	154.74
9	≈ 200

d) $10{,}000 = (92.2657)(1.09)^x$

$108.38 = 1.09^x$

$\log 108.38 = x \log 1.09$

$x \approx 54^{th}$ day