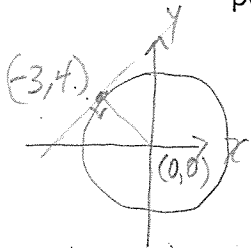


Geometry (H)
Equations of Circles – Problems

Name: KEY

1. Write the equation of the line that is tangent to the circle $x^2 + y^2 = 25$, at the point $(-3, 4)$.



① Find slope of radius.

$$m = \frac{4-0}{-3-0} = -\frac{4}{3}$$

② $M_{\text{tan}} = M_{\perp} = \frac{3}{4}$

③ $y = mx + b$

$$4 = \frac{3}{4}(-3) + b$$

$$\frac{16}{4} + \frac{9}{4} = b$$

$$\frac{25}{4} = b$$

$$y = \frac{3}{4}x + \frac{25}{4}$$

2. Write the equation of the circle that has its center on the line $y = \frac{1}{2}x$ and contains the points $(0, 6)$ and $(0, -2)$. Find (h, k)

EQs: $r^2 = (0-h)^2 + (6-k)^2$

$$h^2 + 36 - 12k + k^2$$

$$r^2 = (0+h)^2 + (-2-k)^2$$

$$h^2 + 4 + 4k + k^2$$

Find (h, k)
 $r^2 = r^2$
 $h^2 + 36 - 12k + k^2 = h^2 + 4 + 4k + k^2$

$$36 - 12k = 4 + 4k$$

$$k = 2 \rightarrow y = 2$$

$$y = \frac{1}{2}x$$

$$2 = \frac{1}{2}x \rightarrow x = 4 \rightarrow h = 4$$

Find radius using $(4, 2)$ & $(0, 6)$
 $d = \sqrt{32} = 4\sqrt{2}$

$$(x-4)^2 + (y-2)^2 = 32$$

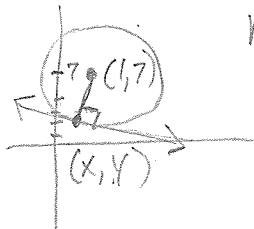
Check by plugging in $(0, 6)$ & $(0, -2)$!

3. Write the equation of the circle with center $(1, 7)$ and tangent to the line $x + 3y = 12$.

$$x + 3y = 12$$

$$3y = -x + 12$$

$$y = -\frac{1}{3}x + 4$$



$M_{\text{radius}} = \frac{3}{4} (x, y)$

$$m = \frac{y-7}{x-1}$$

$$3 = \frac{y-7}{x-1} \rightarrow 3x-3 = y-7$$

$$3x+4 = y$$

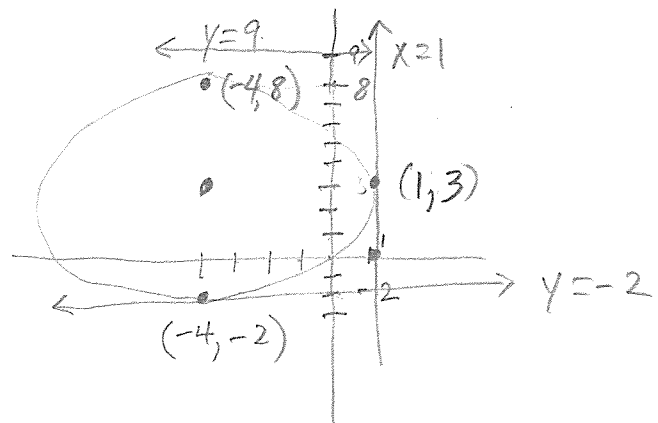
$$3x - y = -4 \rightarrow 9x - 3y = -12$$

$$x + 3y = 12 \rightarrow x + 3y = 12$$

length of radius = $\sqrt{10}$
 $(x-1)^2 + (y-7)^2 = 10$
 $10x = 0$
 $x = 0$
 $y = 4$

4. Given the circle $(x+4)^2 + (y-3)^2 = 25$, determine if each line is a tangent to the circle, secant to the circle or neither.
center $(-4, 3)$ radius = 5

- a. $y = -2$ tangent
- b. $y = -x$ secant
- c. $x = 1$ tangent
- d. $y = 9$ neither



a) $(x+4)^2 = 0$
 $x = -4, y = -2$

b) $x^2 + 8x + 16 + x^2 + 6x + 9 = 25$
 $2x^2 + 14x + 25 = 25$

2 pts work.

$$2x^2 + 14x = 0$$

$$2x(x+7) = 0$$

$$x = 0, x = -7$$

$$x = 0, y = 7$$

c) $y = 3$

d) $169 + (y-3)^2 = 25$
 $(y-3)^2 = -$

5. Write an equation of the form $x^2 + y^2 + dx + ey + f = 0$ for the circle with Center Q and radius of length r.

a. $Q(-3,4), r = 5$

$$(x+3)^2 + (y-4)^2 = 25$$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = 25$$

$$x^2 + y^2 + 6x - 8y = 0$$

b. $Q(4,-2), r = 4$

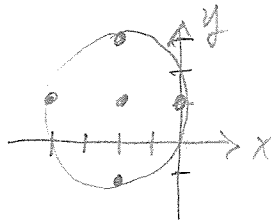
$$(x-4)^2 + (y+2)^2 = 16$$

$$x^2 - 8x + 16 + y^2 + 4y + 4 = 16$$

$$x^2 + y^2 - 8x + 4y + 4 = 0$$

6. Draw the graph for the circle $(x+2)^2 + (y-1)^2 = 4$. State the center and radius.

center $(-2, 1)$
radius = 2



7. Write an equation of the circle containing the two given points as endpoints of a diameter.

(1,2) and (-3,-4)

diameter $\rightarrow d = \sqrt{52} = 2\sqrt{13}$
 $r = \frac{2\sqrt{13}}{2} = \sqrt{13}$

points
midpoint $(\frac{1-3}{2}, \frac{2-4}{2}) = (-1, -1)$
 h, k

$$(x+1)^2 + (y+1)^2 = 13$$

8. Tell whether point A is on the circle with the given equation.

A(2,5), $x^2 + y^2 + 2x - 8y + 13 = 0$

$$2^2 + 5^2 + 2(2) - 8(5) + 13 \stackrel{?}{=} 0$$

$$6 > 0$$

No, point A is NOT on the circle.

If you put Eq in standard form, you'll get
 $(x+1)^2 + (y-4)^2 = 4$
 Plug in (2,5)
 Get $10 > 4$ also.

9. Find the coordinates of the center and the length of the radius.

$$x^2 + y^2 + 4x + 2y + 145 = 0$$

$$x^2 + 4x + \frac{4}{4} + y^2 + 2y + \frac{1}{1} = 145 + 4 + 1$$

$$(x+2)^2 + (y+1)^2 = 150$$

center $(-2, -1)$
radius = $5\sqrt{6}$

$$r = \sqrt{150} = \frac{\sqrt{25 \cdot 6}}{5\sqrt{6}}$$

10. Tell whether the two circles are congruent.

$$x^2 + y^2 - 4x + 2y - 4 = 0$$

$$x^2 + y^2 + 2x - 6y - 6 = 0$$

$$x^2 - 4x + \frac{4}{4} + y^2 + 2y + \frac{1}{1} = 4 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 = 9$$

$$r = 3$$

$$x^2 + 2x + \frac{1}{1} + y^2 - 6y + \frac{9}{9} = 16$$

$$(x+1)^2 + (y-3)^2 = 16$$

$$r = 4$$

No, the 2 circles are NOT congruent.

11. Write an equation of the line that is tangent to the given circle at the given point P.

$$x^2 + y^2 + 10x + 8y + 21 = 0, P(-7, -8)$$

Need center, so find standard form:

$$x^2 + 10x + \frac{25}{25} + y^2 + 8y + \frac{16}{16} = -21 + 25 + 16$$

$$(x+5)^2 + (y+4)^2 = 20$$

center $(-5, -4)$

$$r = 2\sqrt{5} \approx \text{bet } 4 \text{ \& } 5$$

$$M_{\text{radius}} = \frac{-4+8}{-5+7} = \frac{4}{2} = 2$$

$$\text{So } M_{\text{tang}} = -\frac{1}{2}$$

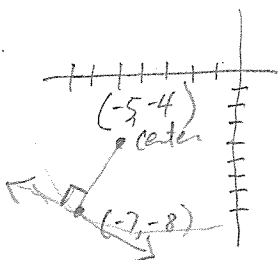
Find y-intercept use P(-7, -8)

$$-8 = -\frac{1}{2}(-7) + b$$

$$-\frac{23}{2} = b$$

$$y = -\frac{1}{2}x - \frac{23}{2}$$

tangent



Need slope of radius in order to get slope of tang.

