# Justifying ASA Triangle Congruence

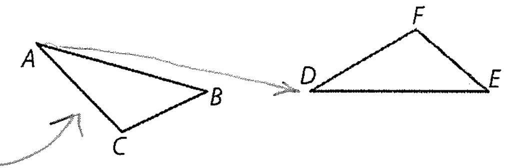
Explain the results of Explore 1 using transformations.

**A** Use tracing paper to make two copies of the triangle from Explore 1 as shown. Identify the corresponding parts you know to be congruent and mark these congruent parts on the figure.

Given: $\angle A \cong$ $\measuredangle D$

$\angle B \cong$ $\measuredangle E$

$\overline{AB} \cong$ $\overline{DE}$

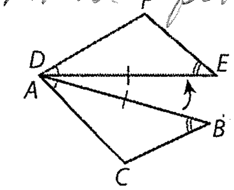



**B** What can you do to show that these triangles are congruent?

Find a sequence of rigid motions that maps one $\triangle$ onto the other $\triangle$.

**C** Translate $\triangle ABC$ so that point $A$ maps to point $D$. What translation vector did you use?

the vector with initial point A and terminal point D ($\overrightarrow{AD}$)



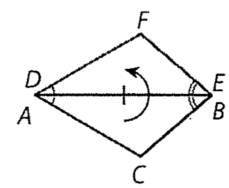

**D** Use a rotation to map point $B$ to point $E$. What is the center of the rotation? What is the angle of the rotation?

Center of rotation is pt. A (or pt. A)

$\measuredangle$ of rotation is $m\measuredangle EDB$.

**E** How do you know the image of point $B$ is point $E$?

It's given that $\overline{AB} \cong \overline{DE}$, so the image of pt B is pt. E.




**F** What rigid motion do you think will map point $C$ to point $F$?

reflection across $\overleftrightarrow{DE}$

**G** To show that the image of point $C$ is point $F$, notice that $\angle A$ is reflected across $\overleftrightarrow{DE}$, so the measure of the angle is preserved. Since $\angle A \cong \angle D$ you can conclude that the image of $\overline{AC}$ lies on $\overrightarrow{DF}$. In particular, the image of point $C$ must lie on $\overrightarrow{DF}$. By similar reasoning, the image of $\overline{BC}$ lies on $\overrightarrow{FE}$ and the image of point $C$ must lie on $\overrightarrow{FE}$. The only point that lies on both $\overline{DF}$ and $\overline{EF}$ is pt. F.

**H** Describe the sequence of rigid motions used to map $\triangle ABC$ to $\triangle DEF$.

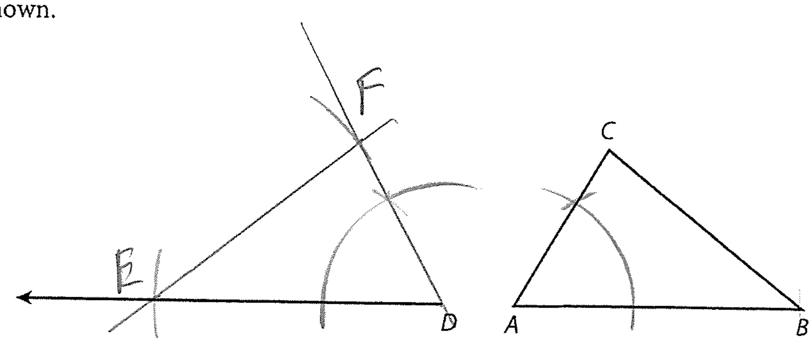
A translation with $\overrightarrow{AD}$ followed by a rotation about $\measuredangle EDB$ followed by a reflection across $\overleftrightarrow{DE}$.

## Explore 2 Justifying SAS Triangle Congruence

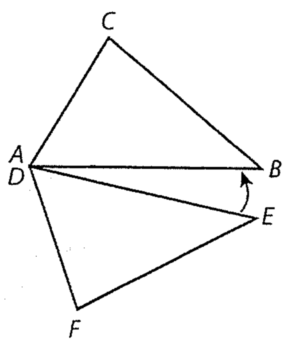
You can explain the results of Explore 1 using transformations.

(A) Construct $\triangle DEF$ by copying $\angle A$, side $\overline{AB}$, and side $\overline{AC}$. Let point $D$ correspond to point $A$, point $E$ correspond to point $B$, and point $F$ correspond to point $C$, and place point $E$ on the segment shown. →

So:
$\angle A \cong \angle D$
$\overline{DE} \cong \overline{AB}$
$\overline{DF} \cong \overline{AC}$




(B) The diagram illustrates one step in a sequence of rigid motions that will map $\triangle DEF$ onto $\triangle ABC$. Describe a complete sequence of rigid motions that will map $\triangle DEF$ onto $\triangle ABC$.




Possible answer: Translate $\triangle DEF$ so that pt D maps to pt A. Then rotate $\triangle DEF$ 180° counterclockwise about pt. A. Pt. E will map to pt. B b/c DE = AB. Then reflect $\triangle DEF$ across $\overline{AD}$. Pt. F will map to pt C b/c $\angle A \cong \angle D$ and DF = AC.

(C) What can you conclude about the relationship between $\triangle ABC$ and $\triangle DEF$? Explain your reasoning.

$\triangle ABC \cong \triangle DEF$ b/c there is a sequence of rigid motions that maps one onto the other.

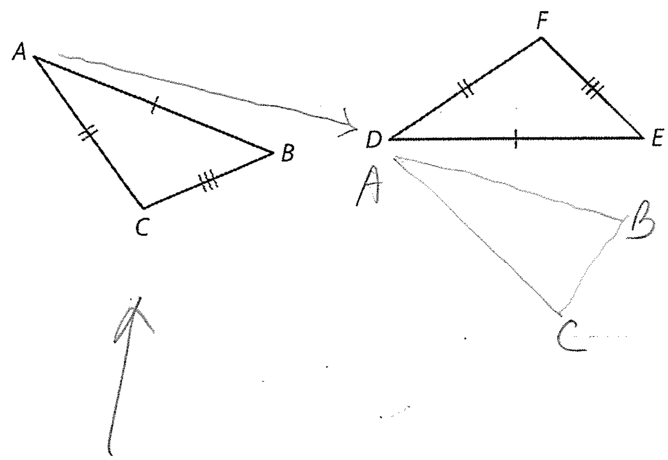
# Justifying SSS Triangle Congruence

You can use rigid motions and the converse of the Perpendicular Bisector Theorem to justify this theorem.

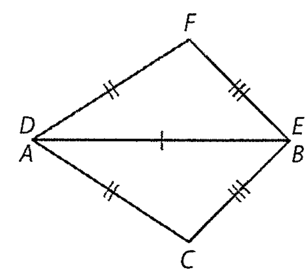
**SSS Triangle Congruence Theorem**

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

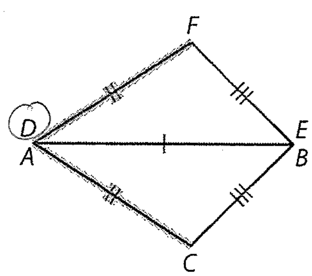
**Example 1** **In the triangles shown, let $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$. Use rigid motions to show that $\triangle ABC \cong \triangle DEF$.**



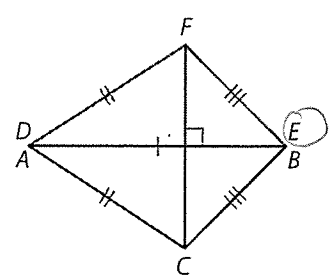

(A) Transform $\triangle ABC$ by a translation along $\overrightarrow{AD}$ followed by a rotation about point $D$, so that $\overline{AB}$ and $\overline{DE}$ coincide. The segments coincide because they are the same length.




Does a reflection across $\overline{AB}$ map point $C$ to point $F$? To show this, notice that $DC = DF$, which means that point $D$ is equidistant from point $C$ and point $F$.

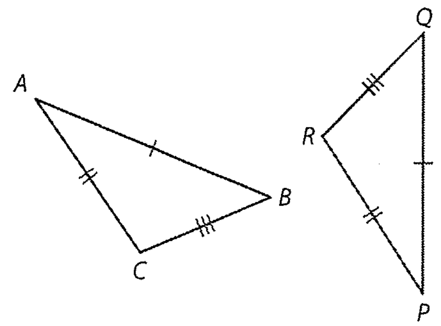



Therefore, point $D$ lies on the perpendicular bisector of $\overline{CF}$ by the converse of the perpendicular bisector theorem. Because $EC = EF$, point $E$ also lies on the perpendicular bisector of $\overline{CF}$.

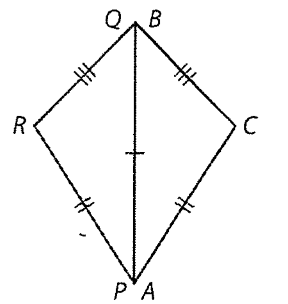



Since point $D$ and point $E$ both lie on the perpendicular bisector of $\overline{CF}$ and there is a unique line through any two points, $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{CF}$. By the definition of reflection, the image of point $C$ must be point $F$. Therefore, $\triangle ABC$ is mapped onto $\triangle DEF$ by a translation, followed by a rotation, followed by a reflection, and the two triangles are congruent.

(B) Show that $\triangle ABC \cong \triangle PQR$.




Triangle *ABC* is transformed by a sequence of rigid motions to form the figure shown below. Identify the sequence of rigid motions. (You will complete the proof on the following page.)

| | |
|---|---|
|   | 1. Translation along $\overrightarrow{AP}$<br>2. Rotation about P so that $\overline{PQ}$ & $\overline{AB}$ coincide.<br>3. Reflection across $\overline{PQ}$. |

Complete the explanation by filling in the blanks with the name of a point, line segment, or geometric theorem.

Because $\overline{QR} \cong$ $\overline{QC}$, point Q is equidistant from R (or C) and C (or R). Therefore, by the converse of the ⊥ bisector Theorem, point Q lies on the ⊥ bisector of $\overline{RC}$. Similarly, $\overline{PR} \cong$ $\overline{PC}$. So point P lies on the perpendicular bisector of $\overline{RC}$. Because two points determine a line, the line $\overleftrightarrow{PQ}$ is the ⊥ bisector of $\overline{RC}$.

By the definition of reflection, the image of point *C* must be point R. Therefore, $\triangle ABC \cong \triangle PQR$ because $\triangle ABC$ is mapped to $\triangle PQR$ by a translation, a rotation, and a reflection.