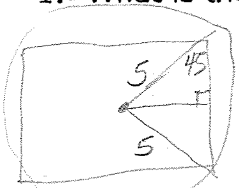
Geometry Honors   Name:

(11.4) Area of Regular Polygons

1. What is the area of a square inscribed in a circle with a radius of 5 cm?



$x\sqrt{2}=5$

$x=\frac{5}{\sqrt{2}}=\frac{5\sqrt{2}}{2}$

Side $=\frac{10\sqrt{2}}{2}=\frac{10}{\sqrt{2}}$

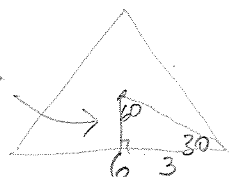
$A=\left(\frac{10}{\sqrt{2}}\right)^2$

$=\frac{100}{2}=$ 50 sq. cm

OR $A=\frac{1}{2}\frac{5}{\sqrt{2}}\frac{40}{\sqrt{2}}$

$=\frac{100}{2}=50$

2. What is the area of an equilateral triangle with side length of 6 cm?



$\frac{3}{\sqrt{3}}$

$x\sqrt{3}=3$

$x=\frac{3}{\sqrt{3}}$

$A=\frac{1}{2}ap$

$=\frac{1}{2}\left(\frac{3}{\sqrt{3}}\right)(18)$

$=\frac{27}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=$ $9\sqrt{3}$ sq. cm

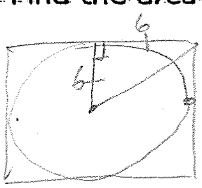
of 1 Small $\triangle$

OR $A=\frac{1}{2}(6)\left(\frac{3}{\sqrt{3}}\right)$

$=\frac{9}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=3\sqrt{3}$

$A_{\triangle}=3\times 3\sqrt{3}=9\sqrt{3}$

3. Find the area of a square circumscribed about a circle with a radius of 6 cm.



Side $=12$

$A=\frac{1}{2}ap$

$=\frac{1}{2}(6)(48)$

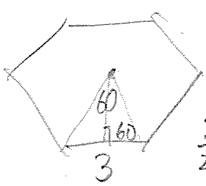
$=144$

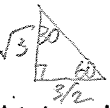
OR $A_{\triangle}=\frac{1}{2}(6)(12)$

$=36$

$A_{\square}=144$

4. What is the area of a regular hexagon with a side length of 3 cm?





$\frac{3\sqrt{3}}{2}$

$\frac{(6-n)180}{6}=120$

$A=\frac{1}{2}\left(\frac{3}{2}\sqrt{3}\right)(18)$

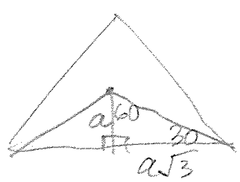
$=\boxed{\frac{27\sqrt{3}}{2}}$

OR $A=\frac{1}{2}\left(\frac{3}{2}\sqrt{3}\right)(3)$

$=\frac{9\sqrt{3}}{4}$

$A=6\left(\frac{9\sqrt{3}}{4}\right)=\boxed{\frac{27\sqrt{3}}{2}}$

5. An equilateral triangle has an area of $24\sqrt{3}$ cm^2. Find the length of each side.



side $=2a\sqrt{3}$

$a\sqrt{3}$

$\frac{1}{2}ap=24\sqrt{3}$

$\frac{1}{2}a(6a\sqrt{3})=24\sqrt{3}$

$3a^2=24$

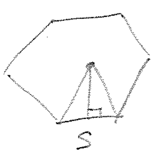
$a^2=8$

$a=2\sqrt{2}$

side $=2a\sqrt{3}=2\cdot 2\sqrt{2}\sqrt{3}$

side $=4\sqrt{6}$

6. The apothem of a regular hexagon is $3\sqrt{3}$. What is the length of the side?



$3\sqrt{3}$

$a=3\sqrt{3}$

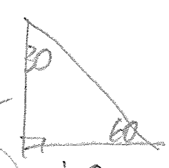
each $\angle=\frac{(6-2)180}{6}$

$=\frac{4}{6}(180)$

$=120$

each side $=6$ units

7. The area of a regular hexagon is $72\sqrt{3}$. What is its side length?



$a=\frac{1}{2}s\sqrt{3}$

$72\sqrt{3}=\frac{1}{2}ap$

$=\frac{1}{2}\left(\frac{\sqrt{3}}{2}s\right)(6s)$

$=\frac{3\sqrt{3}s^2}{2}$

$\frac{72\sqrt{3}}{1}\left(\frac{2}{3\sqrt{3}}\right)=s^2$

$48=s^2$

$4\sqrt{3}=s$

$a=\frac{1}{2}4\sqrt{3}\sqrt{3}=6$

Ck

$72\sqrt{3}=\frac{1}{2}ap$

$=\frac{1}{2}(6)(24\sqrt{3})$

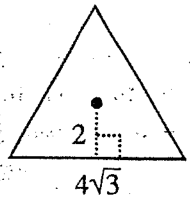
$\checkmark$

$=72\sqrt{3}$

Geometry (H)
Section 11.4 – Area of Regular Polygons

Name: KEY

Find the area of each regular polygon.

1.


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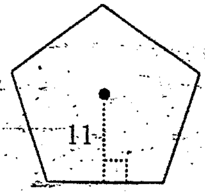
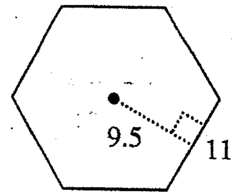
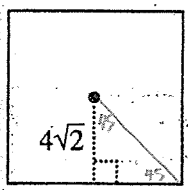
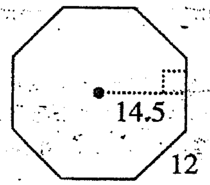
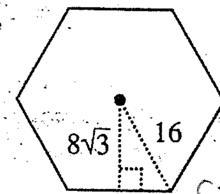


3.


4.


8√2

5.


6.


S=16

① $A=\frac{1}{2}(2)(12\sqrt{3}) = 12\sqrt{3}$

② $A=\frac{1}{2}(11)(80) = 440$

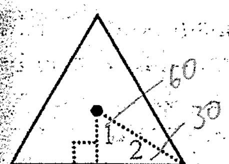
③ $A=\frac{1}{2}(9.5)(66) = 313.5$

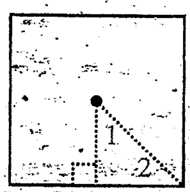
④ $A=\frac{1}{2}(4\sqrt{2})(32\sqrt{2}) = 128$

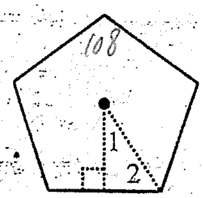
⑤ $A=\frac{1}{2}(14.5)(96) = 696$

⑥ $A=\frac{1}{2}(8\sqrt{3})(96) = 384\sqrt{3}$

Find the measure of ∠1 and ∠2 in each regular polygon.

7.


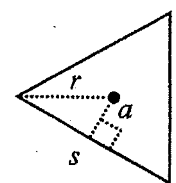
8.


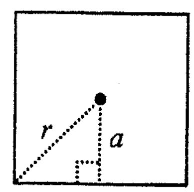
9.


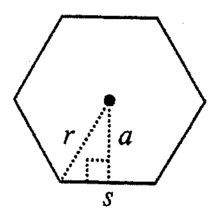
	m∠1	m∠2
⑦	60°	30°
⑧	45°	45°
⑨	36°	54°

Complete the table for the regular polygons to the right.

	Figure	r	s	p	a	A
10.	triangle	12	$12\sqrt{3}$	$36\sqrt{3}$	6	$108\sqrt{3}$
11.	triangle	10	$10\sqrt{3}$	$30\sqrt{3}$	5	$75\sqrt{3}$
12.	triangle	8	$8\sqrt{3}$	$24\sqrt{3}$	4	$48\sqrt{3}$
13.	square	$8\sqrt{2}$	16	64	8	256
14.	square	$8\sqrt{2}$	16	64	a	256
15.	square	10	$10\sqrt{2}$	$40\sqrt{2}$	$5\sqrt{2}$	$100\sqrt{2}$
16.	hexagon	10	10	60	$5\sqrt{3}$	$150\sqrt{3}$
17.	hexagon	18	18	108	$9\sqrt{3}$	$486\sqrt{3}$
18.	hexagon	6	6	36	$3\sqrt{3}$	$54\sqrt{3}$



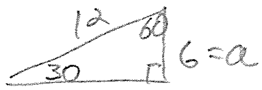




Determine whether each statement is true or false. If false, tell why.

19. An apothem of a regular polygon is a perpendicular bisector of a side.
20. An apothem of a regular polygon is the radius of the circumscribed circle.
21. A radius of a regular polygon bisects an angle of the polygon.
22. A radius of a regular polygon is a radius of the inscribed circle.
23. A radius of a regular polygon is always longer than an apothem.
24. Only regular polygons have apothems.

(10)



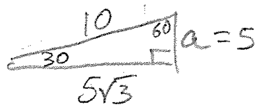
$6\sqrt{3} \times 2 \rightarrow 12\sqrt{3} = S$

$A = \frac{1}{2} a p$

$A = \frac{1}{2} 6 (36\sqrt{3})$

$A = 108\sqrt{3}$ sq. units

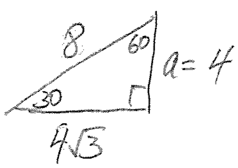
(11)



$A = \frac{1}{2}(5)(30\sqrt{3})$

$A = 75\sqrt{3}$ sq.

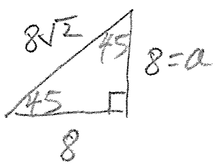
(12)



$A = \frac{1}{2}(4)(24\sqrt{3})$

$= 48\sqrt{3}$ sq.

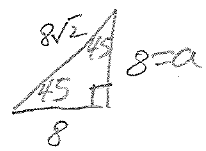
(13)



$A = \frac{1}{2}(8)(64)$

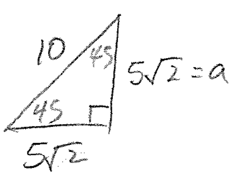
$= 256$ sq.

(14)



$A = \frac{1}{2}(8)(64)$

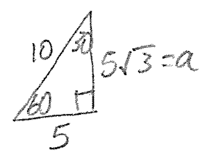
(15)



$A = \frac{1}{2}(5\sqrt{2})(40\sqrt{2})$

$= 100\sqrt{2}$ sq.

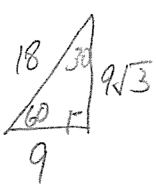
(16)



$A = \frac{1}{2}(5\sqrt{3})(60)$

$= 150\sqrt{3}$ sq.

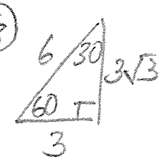
(17)



$A = \frac{1}{2}(9\sqrt{3})(108)$

$= 486\sqrt{3}$

(18)



$A = \frac{1}{2}(3\sqrt{3})(36)$

$= 54\sqrt{3}$ sq.

(19) True

(20) False. The apothem is ⊥ to side of polygon inscribed in a circle. So the apothem does not meet the circumscribed circle

(21) T

(22) False. It is the radius of the circumscribed circle. The radius of the inscribed circle would be shorter.

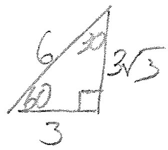
(23) True

(24) True.

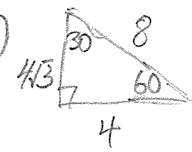
11.4 continued

(25) $A = \frac{1}{2} 3\sqrt{3}(36)$

$= 54\sqrt{3}$ sq



(26)



$A = \frac{1}{2} 4\sqrt{3}(48)$

$= 96\sqrt{3}$ sq.

(27) $A = \frac{1}{2} a p$

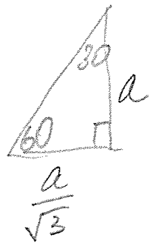
$36\sqrt{3} = \frac{1}{2} a \frac{12a}{\sqrt{3}}$

$36\sqrt{3} = \frac{6a^2}{\sqrt{3}}$

$6 \cdot 3$

$18 = a^2$

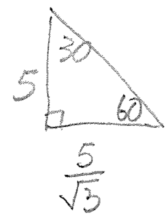
$3\sqrt{2} = a$



(side $= \frac{2a}{\sqrt{3}}$)

side $= 2\sqrt{6}$

(28)



$P = 6 \cdot$ side

$= 6\left(\frac{10}{\sqrt{3}}\right)$

$= \frac{60}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

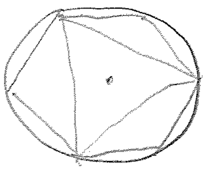
$P = 20\sqrt{3}$

$A = \frac{1}{2} a p$

$= \frac{1}{2} 5(20\sqrt{3})$

$= 50\sqrt{3}$ sq.

(29)

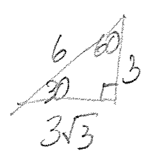


hex

$A = \frac{1}{2}(3\sqrt{3})(36)$

$= 54\sqrt{3}$

triangle



side $= 6\sqrt{3}$

$A = \frac{1}{2}(3)(18\sqrt{3})$

$= 27\sqrt{3}$

$A_{hex} - A_{\Delta} =$ diff

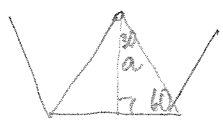
$54\sqrt{3} - 27\sqrt{3} = 27\sqrt{3}$

(30) $A = \frac{1}{2} a p$

$= \frac{1}{2}\left(\frac{\sqrt{3}\, s}{2}\right)(6s)$

$= \boxed{\frac{3s^2\sqrt{3}}{2}}$

(31)



$2\sqrt{a}$

side $= 2\sqrt{a}$

side $= \frac{2a\sqrt{3}}{3}$

$2\sqrt{a} = \frac{2a\sqrt{3}}{3}$

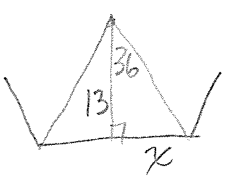
$(3\sqrt{a})^2 = (a\sqrt{3})^2$

$9a = a^2 \cdot 3$

$3 = a$

side $= 2\sqrt{3}$

(32)



$\tan 36 = \frac{x}{13}$

$x = 13 \tan 36$

side $= 26 \tan 36$

$p = 130 \tan 36$

$A = \frac{1}{2}(13)(130 \tan 36)$

$= 613.9$ sq. units

(33) a = apothem

$p = 3 + 7a$

$A = \frac{1}{2} a p$

$440 = \frac{1}{2} a(3 + 7a)$

$880 = 3a + 7a^2$

$0 = 7a^2 + 3a - 880$

$0 = (7a + 80)(a - 11)$

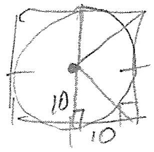
OMIT $a = \frac{-80}{7}$ $\quad a = 11$

$a = 11$

$p = 3 + 77 = 80$

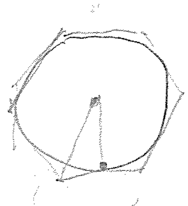
side $= \frac{80}{5} = 16$

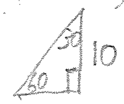
(34)



$S = 20$

$A = 400$





$\frac{10\sqrt{3}}{3}$ side $= \frac{20\sqrt{3}}{3}$

$p = 40\sqrt{3}$

$A_{sq} - A_{hex} = A_{extra}$

$\boxed{400 - 200\sqrt{3} = A_{extra}}$

$A_{hex} = \frac{1}{2}(10)(40\sqrt{3})$
$= 200\sqrt{3}$

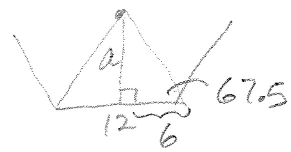
(35)

$A_{border} = A_{OCT} - A_{SQ}$
$= 695.29 - 419.65$
$\approx \boxed{275.64 \text{ sq.}}$

OR

$\boxed{144\sqrt{2} + 72}$

Octagon



$\tan 67.5 = \frac{a}{6}$

$a = 6 \tan 67.5$

$A = \frac{1}{2}(6 \tan 67.5)(96)$

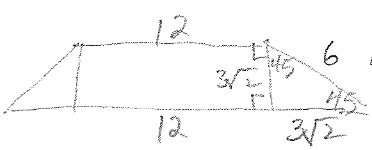
$A = 695.293$

SQ

diagonal $= 2a$
$= 2(6 \tan 67.5)$
$= 12 \tan 67.5$

$A = \frac{1}{2} d_1 d_2 = \frac{1}{2}(12 \tan 67.5)^2$
$= 419.647$

OR

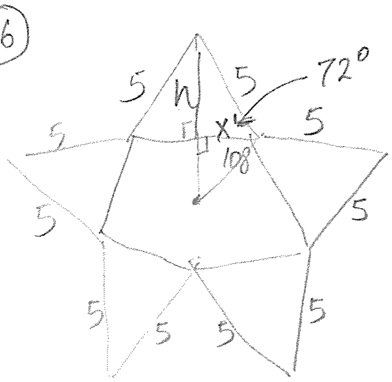


$A_{border} = 2 \cdot A_{\Delta} + A_{rect.}$
$= 2(\frac{1}{2})(3\sqrt{2})(3\sqrt{2}) + 12(3\sqrt{2})$

$9 \cdot 2 + 36\sqrt{2}$

$4 \cdot A_{border} = 4(18 + 36\sqrt{2})$
$= \boxed{144\sqrt{2} + 72}$

(36)



$\sin 72 = \frac{a}{5}$

$h = 5 \sin 72$

$h \approx 4.8$

$\cos 72 = \frac{x}{5}$

$x = 5 \cos 72$

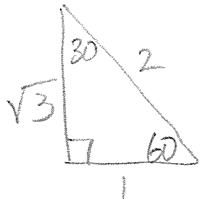
$x \approx 1.5$

$A_{\Delta} = \frac{1}{2}(3)(4.8) = 7.2 \rightarrow A_{5\Delta s} = 36$

$A_{pent} = 15.75$

$\boxed{A_{star} = 51.75}$

37



$A = \frac{1}{2}(\sqrt{3})(12) = 6\sqrt{3}$ sq feet

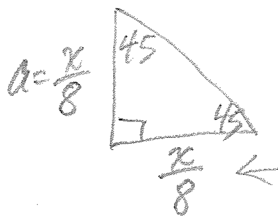
$\$2(6\sqrt{3}) \approx \$20.78$

38

square

Let x = perimeter

$\frac{x}{4}$ = a side



$\leftarrow \left(\frac{x}{4} \cdot \frac{1}{2}\right)$

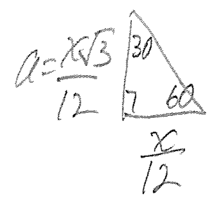
$A = \frac{1}{2}\left(\frac{x}{8}\right)x$

$A = \frac{x^2}{16}$

hexagon

x = perimeter

$\frac{x}{6}$ = a side



$\leftarrow \left(\frac{x}{6} \cdot \frac{1}{2}\right)$

$A = \frac{1}{2}\left(\frac{x\sqrt{3}}{12}\right)x$

$A = \frac{x^2\sqrt{3}}{24}$

$$\frac{x^2}{16} : \frac{x^2\sqrt{3}}{24}$$

$$(16)\ \frac{1}{16} : \frac{\sqrt{3}}{24}\ (16)$$

$$1 : \frac{2\sqrt{3}}{3}$$

The area of the hexagon is $\frac{2\sqrt{3}}{3}$ times the area of the square.

(39)

$\triangle$

Let x = perimeter

$\frac{x}{3}$ = side

$A = \frac{1}{2}\left(\frac{x\sqrt{3}}{18}\right)(x)$

hex

x = perimeter

$\frac{x}{6}$ = side

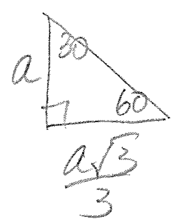
$A = \frac{1}{2}\left(\frac{x\sqrt{3}}{12}\right)x$

$$\frac{x^2\sqrt{3}}{18} : \frac{x^2\sqrt{3}}{12}$$

$$(36)\,\frac{1}{18} : \frac{1}{12}\,(36)$$

$$2 : 3$$

(40)



side $= \frac{2a\sqrt{3}}{3}$

$p = 4a\sqrt{3}$

$50\sqrt{3} = \frac{1}{2}a(4a\sqrt{3})$

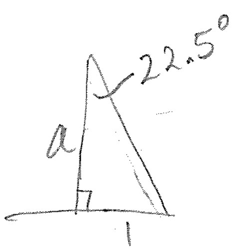
$50 = 2a^2$

$25 = a^2$

5 = apoth.

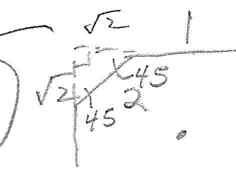
perimeter $= 20\sqrt{3}$

(41)

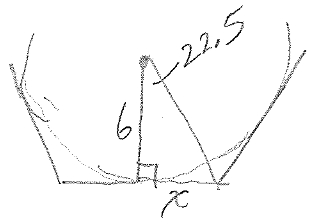


$\tan 22.5 = \frac{1}{a}$

$a \approx 2.4$ units or $1+\sqrt{2}$



(42)



$\tan 22.5 = \frac{x}{6}$

$x = 6\tan 22.5$

Circumscribed Octo

$a = 6$

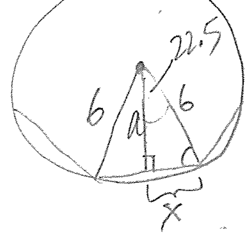
side $= 12\tan 22.5$

$p = 96\tan 22.5$

$A = \frac{1}{2}(6)(96\tan 22.5)$

$= 119.3$

Incribed



$\cos 22.5 = \frac{a}{6}$

$a = 6\cos 22.5$

$\sin 22.5 = \frac{x}{6}$

$x = 6\sin 22.5$

side $= 12\sin 22.5$

$p = 96\sin 22.5$

$A = 101.8$

Difference = 17.5 sq.

(43)

$A_{OCT} = \frac{1}{2}r(8s)$ ← perimeter

$= 4rs$

$\rightarrow A_{\text{doubled radius}} = \frac{1}{2}2r(16s)$

$= 16rs$

The area is multiplied by 4.