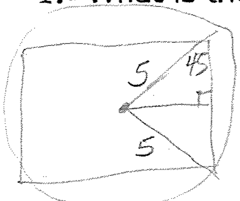
Geometry Honors
~~(11.4)~~ Area of Regular Polygons

Name:

1. What is the area of a square inscribed in a circle with a radius of 5 cm?



$x\sqrt{2} = 5$

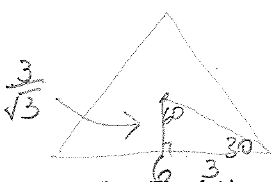
$x = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

$Side = \frac{10\sqrt{2}}{2} = \frac{10}{\sqrt{2}}$

$A = \left(\frac{10}{\sqrt{2}}\right)^2 = \frac{100}{2} = 50$ sq. cm

OR $A = \frac{1}{2}\frac{5}{\sqrt{2}}\frac{40}{\sqrt{2}} = \frac{100}{2} = 50$

2. What is the area of an equilateral triangle with side length of 6 cm?



$x\sqrt{3} = 3$

$x = \frac{3}{\sqrt{3}}$

$A = \frac{1}{2}ap$

$= \frac{1}{2}\left(\frac{3}{\sqrt{3}}\right)(18)$

$= \frac{27}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 9\sqrt{3}$ sq. cm

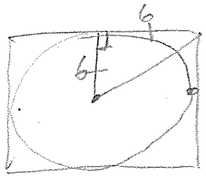
of 1 small Δ

OR $A = \frac{1}{2}(6)\left(\frac{3}{\sqrt{3}}\right)$

$= \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 3\sqrt{3}$

$A_{\Delta} = 3 \times 3\sqrt{3} = 9\sqrt{3}$

3. Find the area of a square circumscribed about a circle with a radius of 6 cm.



Side $= 12$

$A = \frac{1}{2}ap$

$= \frac{1}{2}(6)(48)$

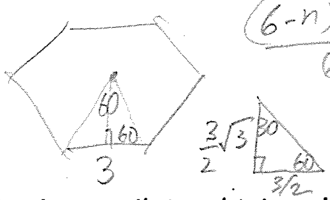
$= 144$

OR $A_{\Delta} = \frac{1}{2}(6)(12)$

$= 36$

$A_{\square} = 144$

4. What is the area of a regular hexagon with a side length of 3 cm?



$\frac{(6-n)180}{6} = 120$

$A = \frac{1}{2}\left(\frac{3}{2}\sqrt{3}\right)(18)$

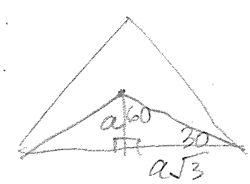
$= \boxed{\frac{27\sqrt{3}}{2}}$

OR $A = \frac{1}{2}\left(\frac{3}{2}\sqrt{3}\right)(3)$

$= \frac{9\sqrt{3}}{4}$

$A = 6\left(\frac{9\sqrt{3}}{4}\right) = \boxed{\frac{27\sqrt{3}}{2}}$

5. An equilateral triangle has an area of $24\sqrt{3}$ cm^2. Find the length of each side.



side $= 2a\sqrt{3}$

$\frac{1}{2}ap = 24\sqrt{3}$

$\frac{1}{2}a(6a\sqrt{3}) = 24\sqrt{3}$

$3a^2 = 24$

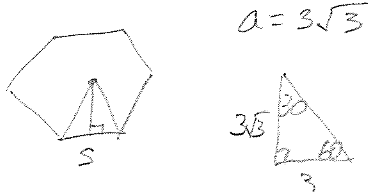
$a^2 = 8$

$a = 2\sqrt{2}$

side $= 2a\sqrt{3} = 2 \cdot 2\sqrt{2}\sqrt{3}$

side $= 4\sqrt{6}$

6. The apothem of a regular hexagon is $3\sqrt{3}$. What is the length of the side?



$a = 3\sqrt{3}$

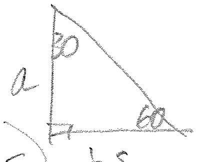
each $\angle = \frac{(6-2)180}{6}$

$= \frac{4}{6}(180)$

$= 120$

each side $= 6$ units

7. The area of a regular hexagon is $72\sqrt{3}$. What is its side length?



$\left(a = \frac{1}{2}s\sqrt{3}\right)$

$72\sqrt{3} = \frac{1}{2}ap$

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2}s\right)(6s)$

$= \frac{3\sqrt{3}s^2}{2}$

$\frac{72\sqrt{3}}{1}\left(\frac{2}{3\sqrt{3}}\right) = s^2$

$48 = s^2$

$4\sqrt{3} = s$

$a = \frac{1}{2}4\sqrt{3}\sqrt{3} = 6$

ck

$72\sqrt{3} = \frac{1}{2}ap$

$= \frac{1}{2}(6)(24\sqrt{3})$

$= 72\sqrt{3}$ ✓

Areas of Regular Polygons

(10)

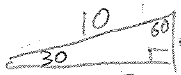
$6\sqrt{3} \times 2 \rightarrow 12\sqrt{3} = S$

$A = \frac{1}{2} a p$

$A = \frac{1}{2} 6 (36\sqrt{3})$

$A = 108\sqrt{3}$ sq. units

(11)

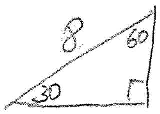


$5\sqrt{3}$

$A = \frac{1}{2}(5)(30\sqrt{3})$

$A = 75\sqrt{3}$ sq.

(12)

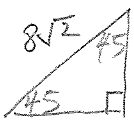


$4\sqrt{3}$

$A = \frac{1}{2}(4)(24\sqrt{3})$

$= 48\sqrt{3}$ sq.

(13)



8

$A = \frac{1}{2}(8)(64)$

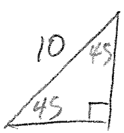
$= 256$ sq.

(14)

8

$A = \frac{1}{2}(8)(64)$

(15)



$5\sqrt{2}$

$A = \frac{1}{2}(5\sqrt{2})(40\sqrt{2})$

$= 100\sqrt{2}$ sq.

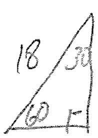
(16)

5

$A = \frac{1}{2}(5\sqrt{3})(60)$

$= 150\sqrt{3}$ sq.

(17)

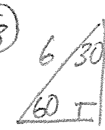


9

$A = \frac{1}{2}(9\sqrt{3})(108)$

$= 486\sqrt{3}$

(18)



3

$A = \frac{1}{2}(3\sqrt{3})(36)$

$= 54\sqrt{3}$ sq.

(19) True (20) False. The apothem is $\perp$ to side of polygon inscribed in a circle. So the apothem does not meet the circumscribed circle

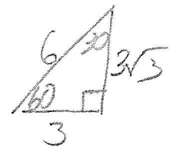
(21) T (22) False. It is the radius of the circumscribed circle. The radius of the inscribed circle would be shorter.

(23) True (24) True.

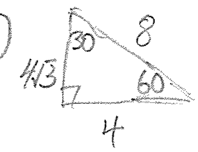
11.4 continued

(25) $A = \frac{1}{2} 3\sqrt{3}(36)$

$= 54\sqrt{3}$ sq



(26)



$A = \frac{1}{2} 4\sqrt{3}(48)$

$= 96\sqrt{3}$ sq.

(27) $A = \frac{1}{2} ap$

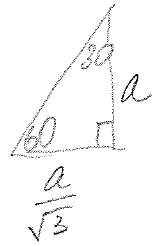
$36\sqrt{3} = \frac{1}{2} a \frac{12a}{\sqrt{3}}$

$36\sqrt{3} = \frac{6a^2}{\sqrt{3}}$

$6 \cdot 3$

$18 = a^2$

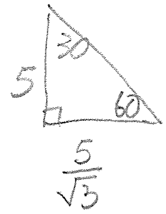
$3\sqrt{2} = a$



$(\text{side} = \frac{2a}{\sqrt{3}})$

side $= 2\sqrt{6}$

(28)



$P = 6 \cdot \text{side}$

$= 6\left(\frac{10}{\sqrt{3}}\right)$

$= \frac{60}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

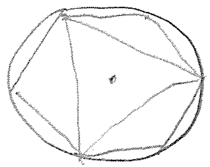
$P = 20\sqrt{3}$

$A = \frac{1}{2} ap$

$= \frac{1}{2} 5(20\sqrt{3})$

$= 50\sqrt{3}$ sq.

(29)

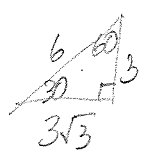


hex

$A = \frac{1}{2}(3\sqrt{3})(36)$

$= 54\sqrt{3}$

triangle



side $= 6\sqrt{3}$

$A = \frac{1}{2}(3)(18\sqrt{3})$

$= 27\sqrt{3}$

$A_{hex} - A_{\Delta} = \text{diff}$

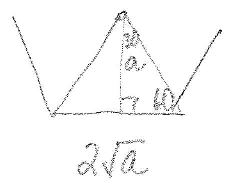
$54\sqrt{3} - 27\sqrt{3} = 27\sqrt{3}$

(30) $A = \frac{1}{2} ap$

$= \frac{1}{2}\left(\frac{\sqrt{3}\,s}{2}\right)(6s)$

$= \boxed{\frac{3s^2\sqrt{3}}{2}}$

(31)



side $= 2\sqrt{a}$

side $= \frac{2a\sqrt{3}}{3}$

$2\sqrt{a} = \frac{2a\sqrt{3}}{3}$

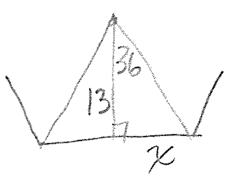
$(3\sqrt{a})^2 = (a\sqrt{3})^2$

$9a = a^2 \cdot 3$

$3 = a$

side $= 2\sqrt{3}$

(32)



$\tan 36 = \frac{x}{13}$

$x = 13 \tan 36$

side $= 26 \tan 36$

$p = 130 \tan 36$

$A = \frac{1}{2}(13)(130 \tan 36)$

$= \boxed{613.9 \text{ sq. units}}$

(33) a: apothem

$p = 3 + 7a$

$A = \frac{1}{2} ap$

$440 = \frac{1}{2} a(3 + 7a)$

$880 = 3a + 7a^2$

$0 = 7a^2 + 3a - 880$

$0 = (7a + 80)(a - 11)$

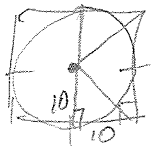
OMIT $a = \frac{-80}{7}$ $a = 11$

$a = 11$

$p = 3 + 77 = 80$

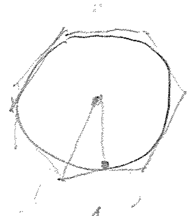
side $= \frac{80}{5} = 16$

(34)



$S = 20$

$A = 400$



$\frac{10\sqrt{3}}{3}$ side $= \frac{20\sqrt{3}}{3}$

$p = 40\sqrt{3}$

$A_{sq} - A_{hex} = A_{extra}$

$\boxed{400 - 200\sqrt{3} = A_{extra}}$

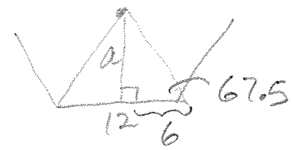
$A_{hex} = \frac{1}{2}(10)(40\sqrt{3})$
$= 200\sqrt{3}$

(35) $A_{border} = A_{OCT} - A_{SQ}$
$= 695.29 - 419.65$
$\approx \boxed{275.64 \text{ sq.}}$

OR

$\boxed{144\sqrt{2} + 72}$

Octagon



$\tan 67.5 = \frac{a}{6}$

$a = 6 \tan 67.5$

$A = \frac{1}{2}(6 \tan 67.5)(96)$

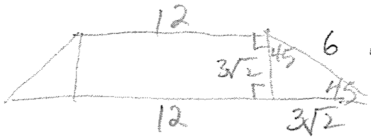
$A = 695.293$

SQ

diagonal $= 2a$
$= 2(6 \tan 67.5)$
$= 12 \tan 67.5$

$A = \frac{1}{2} d_1 d_2 = \frac{1}{2}(12 \tan 67.5)^2$
$= 419.647$

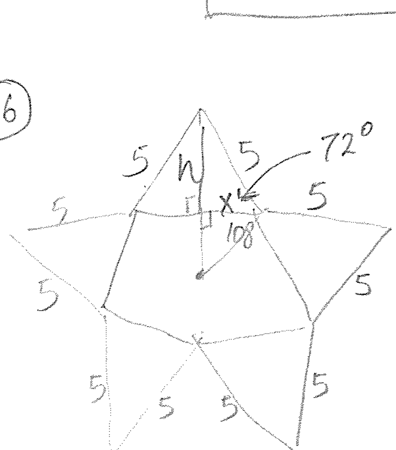
OR



$A_{border} = 2 \cdot A_{\Delta} + A_{rect.}$
$= 2(\frac{1}{2})(3\sqrt{2})(3\sqrt{2}) + 12(3\sqrt{2})$
$9 \cdot 2 + 36\sqrt{2}$

$4 \cdot A_{border} = 4(18 + 36\sqrt{2})$
$= \boxed{144\sqrt{2} + 72}$

(36)



$\sin 72 = \frac{a}{5}$ $\quad$ $\cos 72 = \frac{x}{5}$

$h = 5 \sin 72$ $\quad$ $x = 5 \cos 72$

$h \approx 4.8$ $\quad$ $x \approx 1.5$

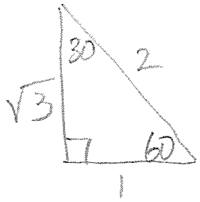
$A_{\Delta} = \frac{1}{2}(3)(4.8) = 7.2 \rightarrow A_{5\Delta s} = 36$

$A_{pent} = 15.75$

$\boxed{A_{STAR} = 51.75}$

(11.4 HW)

37



$A = \frac{1}{2}(\sqrt{3})(12) = 6\sqrt{3}$ sq feet

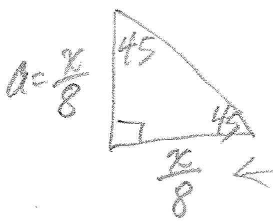
$\$2(6\sqrt{3}) \approx \$20.78$

38

square

Let x = perimeter

$\frac{x}{4}$ = a side



$\leftarrow \left(\frac{x}{4} \cdot \frac{1}{2}\right)$

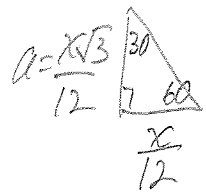
$A = \frac{1}{2}\left(\frac{x}{8}\right)x$

$A = \frac{x^2}{16}$

hexagon

x = perimeter

$\frac{x}{6}$ = a side



$\leftarrow \left(\frac{x}{6} \cdot \frac{1}{2}\right)$

$A = \frac{1}{2}\left(\frac{x\sqrt{3}}{12}\right)x$

$A = \frac{x^2\sqrt{3}}{24}$

$$\frac{x^2}{16} : \frac{x^2\sqrt{3}}{24}$$

$$(16)\,\frac{1}{16} : \frac{\sqrt{3}}{24}\,(16)$$

$$1 : \frac{2\sqrt{3}}{3}$$

The area of the hexagon is $\frac{2\sqrt{3}}{3}$ times the area of the square.

(39)

△

Let x = perimeter

$\frac{x}{3}$ = side

$A = \frac{1}{2}\left(\frac{x\sqrt{3}}{18}\right)(x)$

hex

x = perimeter

$\frac{x}{6}$ = side

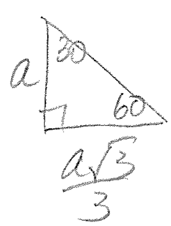
$A = \frac{1}{2}\left(\frac{x\sqrt{3}}{12}\right)x$

$$\frac{x^2\sqrt{3}}{18} : \frac{x^2\sqrt{3}}{12}$$

$$(36)\ \frac{1}{18} : \frac{1}{12}\ (36)$$

$$2 : 3$$

(40)



$\text{side} = \frac{2a\sqrt{3}}{3}$

$p = 4a\sqrt{3}$

$50\sqrt{3} = \frac{1}{2}a(4a\sqrt{3})$

$50 = 2a^2$

$25 = a^2$

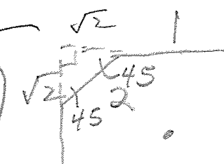
5 = apoth.

perimeter $= 20\sqrt{3}$

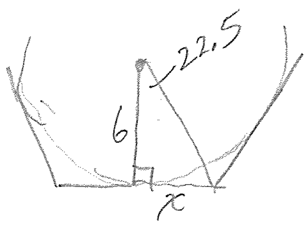
(41)

$\tan 22.5 = \frac{1}{a}$

$a \approx 2.4$ units or $1+\sqrt{2}$



(42)



$\tan 22.5 = \frac{x}{6}$

$x = 6\tan 22.5$

Circumscribed Oct.

$a = 6$

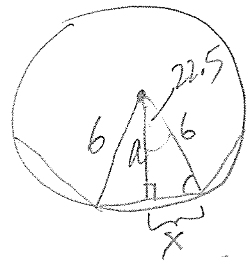
side $= 12\tan 22.5$

$p = 96\tan 22.5$

$A = \frac{1}{2}(6)(96\tan 22.5)$

$= 119.3$

Incribed



$\cos 22.5 = \frac{a}{6}$

$a = 6\cos 22.5$

$\sin 22.5 = \frac{x}{6}$

$x = 6\sin 22.5$

side $= 12\sin 22.5$

$p = 96\sin 22.5$

$A = 101.8$

Difference = 17.5 sq.

(43) $A_{OCT} = \frac{1}{2}r(8s)$ ← perimeter

$= 4rs$

$\rightarrow A_{\text{doubled radius}} = \frac{1}{2}2r(16s)$

$= 16rs$

The area is multiplied by 4.