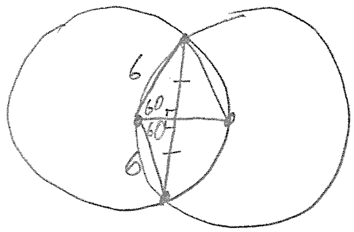
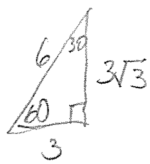
Areas of Sectors - More Problems

(1)



r = 6cm



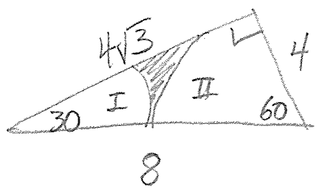
$2\left(A_{sector} - A_{\Delta}\right) = A_{shaded}$

$2\left(\frac{120}{360}\cdot \pi 6^2 - \frac{1}{2}\cdot 6\sqrt{3}\cdot 3\right) =$

$2\left(\frac{1}{3}\cdot 36\pi - 9\sqrt{3}\right)$

$\boxed{24\pi - 18\sqrt{3}} = A_{shaded}$

(2)



$A_{\Delta} - A_{sector\ I} - A_{sector\ II} = A_{shaded}$

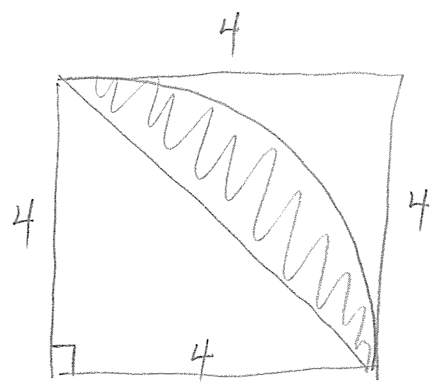
$\frac{1}{2}(4)(4\sqrt{3}) - \frac{30}{360}\pi 4^2 - \frac{60}{360}\pi 4^2$

$8\sqrt{3} - \frac{16\pi}{12} - \frac{1}{6}16\pi$

$8\sqrt{3} - \frac{4\pi}{3} - \frac{8\pi}{3} =$

$\boxed{8\sqrt{3} - 4\pi} = A_{shaded}$

(3)



$A_{sector} - A_{\Delta} = A_{\frac{1}{2}leaf}$

$\frac{1}{4}(\pi 4^2) - \frac{1}{2}(4)(4) =$

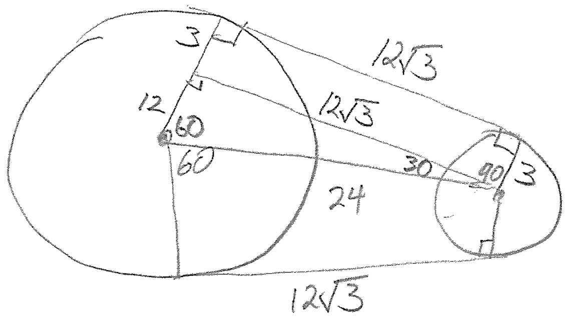
$4\pi - 8 = A_{\frac{1}{2}leaf}$

$\downarrow$

$\boxed{8\pi - 16 = A_{leaf}}$

i

4



$$\text{belt} = 2(12\sqrt{3}) + \begin{matrix}\text{sm}\\ \text{arc}\end{matrix} + \begin{matrix}\text{large}\\ \text{arc}\end{matrix}$$

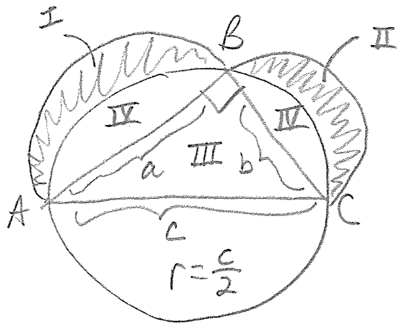
$$= 24\sqrt{3} + \frac{120}{360}2\pi 3 + \frac{240}{360}2\pi 15$$

$$= 24\sqrt{3} + \frac{1}{3}\cdot 6\pi + \frac{2}{3}30\pi$$

$$= 24\sqrt{3} + 2\pi + 20\pi$$

$$= \boxed{24\sqrt{3} + 22\pi}$$

5



Show: $A_{I} + A_{II} = A_{III}$

$$A_{\frown AB} = \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = \frac{a^2\pi}{8}$$

$$A_{\frown BC} = \frac{1}{2}\pi\left(\frac{b}{2}\right)^2 = \frac{b^2\pi}{8}$$

$$A_{III} = \frac{1}{2}ab$$

$$A_{semi} = \frac{1}{2}\pi\left(\frac{c}{2}\right)^2 = \frac{c^2\pi}{8}$$

$$A_{IV} = A_{semi} - A_{III}$$
$$= \left(\frac{c^2\pi}{8} - \frac{1}{2}ab\right)$$

$$A_{I} + A_{II} = A_{\frown AB} + A_{\frown BC} - A_{IV}$$
$$= \frac{a^2\pi}{8} + \frac{b^2\pi}{8} - \left(\frac{c^2\pi}{8} - \frac{1}{2}ab\right)$$
$$= \frac{a^2\pi}{8} + \frac{b^2\pi}{8} - \frac{c^2\pi}{8} + \frac{1}{2}ab$$
$$= \frac{\pi}{8}(a^2+b^2-c^2) + \frac{1}{2}ab$$

Pyth. Thm

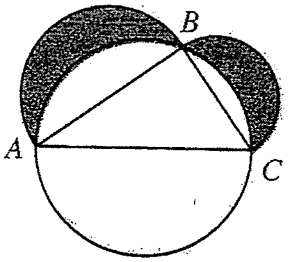
$$\underbrace{a^2+b^2}=c^2$$

$$\frac{\pi}{8}(c^2-c^2) + \frac{1}{2}ab$$

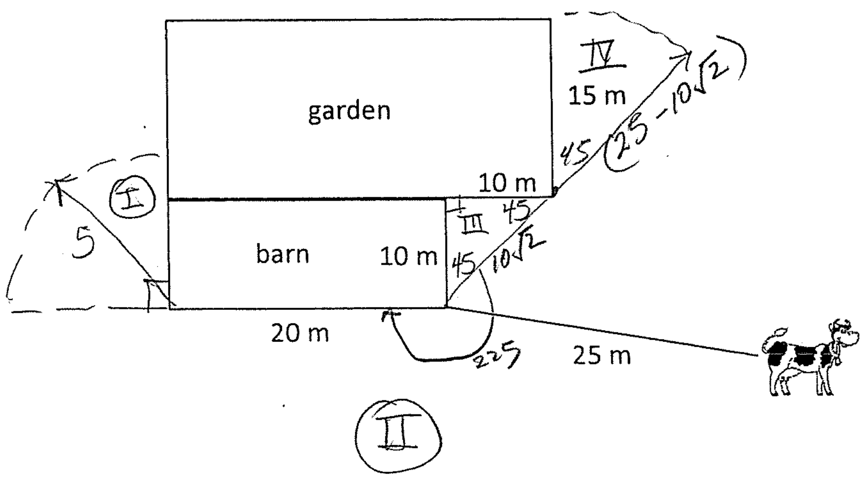
$$A_{I} + A_{II} = \frac{1}{2}ab$$
$$A_{III} = \frac{1}{2}ab$$

equal!

5. Given a right ΔABC, its circumscribed circle, and semicircles on the two legs, show that the sum of the areas of the two shaded regions is equal to the area of ΔABC.



6. A cow is tied by a 25 m rope to the corner of a barn as shown. A fence keeps the cow out of the garden: find to the nearest square meter, the grazing area.
Use $\sqrt{2}$ = 1.414 and π = 3.14



$A_{I} = \frac{1}{4}\pi 5^2 = \frac{25\pi}{4}$

$A_{II} = \frac{225}{360}\pi 25^2 = 390.625\pi$

$A_{III} = \frac{1}{2}(10)(10) = 50$

$A_{IV} = \frac{45}{360}\pi(25-10\sqrt{2})^2 = \frac{1}{8}\pi(117.9396) \approx 46.29129$

$A_{grazing} = \frac{25\pi}{4} + 390.625\pi + 50 + 46.29129$

$\approx \boxed{1{,}342.5 \text{ sq. meters}} \rightarrow \boxed{1343 \text{ sq. Meters}}$ *