

Objective:

To prove lines are parallel

Postulate: Write the converse of the postulate in the previous lesson.

If 2 lines are cut by a transversal, and a pair of same side interior angles are supplementary, then the lines are parallel.

Must prove theorems using the postulate.

Theorem:

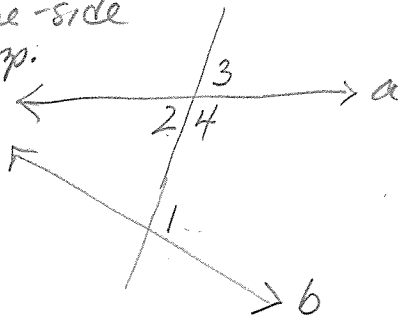
If 2 lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Plan: show same-side int. \angle s supp.

Proof:

Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$



- ① $\angle 1 \cong \angle 2$ —————→ ② $m\angle 1 = m\angle 2$
- ③ $\angle 2$ & $\angle 4$ → ④ $\angle 2$ supp. $\angle 4$ → ⑤ $m\angle 2 + m\angle 4 = 180$ } → ⑥ $m\angle 1 + m\angle 4 = 180$
- linear pair
- ⑦ $\angle 1$ supp. $\angle 4$ → ⑧ $a \parallel b$

- ① Given
- ② $\cong \angle$ s have = measures.
- ③ A linear pair are 2 \angle s that form a straight line
- ④ A linear pair are 2 \angle s supplementary
- ⑤ Suppl. \angle s are 2 \angle s total 180.
- ⑥ Substitution Prop.
- ⑦ Suppl. \angle s are 2 \angle s total 180
- ⑧ 2 lines w/ transv. & same side int. \angle s suppl. → 2 \parallel lines.

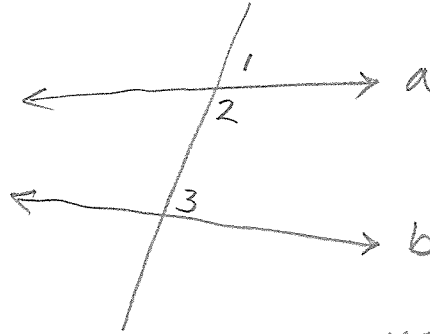
Theorem:

If 2 lines are cut by a transversal and corresponding angles are congruent, then then lines are parallel.

Proof:

Given: $\angle 1 \cong \angle 3$

Prove: $a \parallel b$



- ① $\angle 1 \cong \angle 3$ → ② $m\angle 1 = m\angle 3$
- ③ $\angle 1$ & $\angle 2$ linear pair → ④ $\angle 1$ suppl. $\angle 2$ → ⑤ $m\angle 1 + m\angle 2 = 180$
- ⑥ $m\angle 3 + m\angle 2 = 180$ → ⑦ $\angle 3$ & $\angle 2$ suppl. → ⑧ $a \parallel b$

- ① Given
- ② \cong \angle s have = measures.
- ③ Linear pair are 2 \angle s that form straight line.
- ④ linear pair are 2 \angle s supplementary.
- ⑤ Suppl. \angle s are 2 \angle s total 180.
- ⑥ Substitution Prop.
- ⑦ Suppl. \angle s are 2 \angle s that total 180.
- ⑧ 2 lines w/ trans. & same side int. \angle s suppl. → 2 \parallel lines.

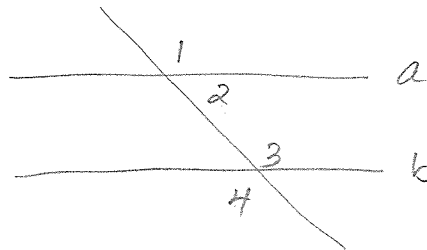
Theorem:

If 2 lines are cut by a transversal and alternate exterior angles are congruent, then then lines are parallel.

Proof:

Given: $\angle 1 \cong \angle 4$

Prove: $a \parallel b$



- ① $\angle 1 \cong \angle 4$ → ② $m\angle 1 = m\angle 4$
- ③ $\angle 1$ & $\angle 2$ linear pair → ④ $\angle 1$ Suppl $\angle 2$ → ⑤ $m\angle 1 + m\angle 2 = 180$
- ⑥ $m\angle 2 + m\angle 4 = 180$ → ⑦ $m\angle 2 + m\angle 3 = 180$
- ⑦ $\angle 3 \cong \angle 4$ → ⑧ $m\angle 3 = m\angle 4$

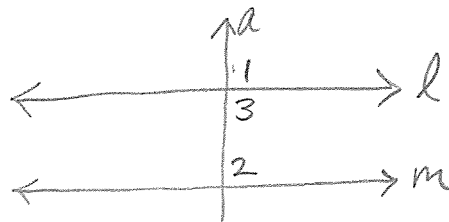
⑩ $\angle 2$ suppl. $\angle 3$ → ⑪ $a \parallel b$

- ① Given
- ② \cong \angle s have = measures.
- ③ linear pair are 2 \angle s that form straight line.
- ④ linear pair are 2 suppl. \angle s.
- ⑤ Suppl. \angle s are 2 \angle s that total 180.
- ⑥ Substitution Prop.
- ⑦ Vertical \angle s \cong .
- ⑧ \cong \angle s have = measures.
- ⑨ Substitution Prop.
- ⑩ Suppl. \angle s are 2 \angle s that total 180.
- ⑪ 2 lines w/ same side int. \angle s suppl → 2 \parallel lines

Theorem:

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one also.

Given: $l \parallel m$
(known) $a \perp l$

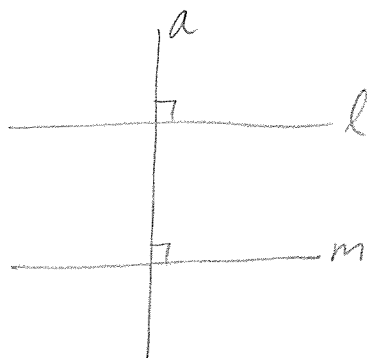


Conclude: $a \perp m$

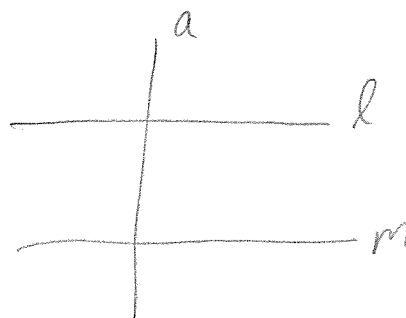
Theorem:

In a plane, two lines perpendicular to the same line are parallel.

Given $l \perp a$
 $m \perp a$



Conclude: $l \parallel m$



The first two theorems useful for drawing lines in diagrams:

- Through a point outside a line, there is exactly one line parallel to the given line.

Given

- Through a point outside a line, there is exactly one line perpendicular to the given line.

Given

- Two lines parallel to a third line are parallel to each other.

Given

The two postulates are useful for drawing lines in diagrams:

- Parallel Postulate: (Thm in orange book)
Through a point outside a line, there is exactly one line parallel to the given line.

Given

$P \cdot$



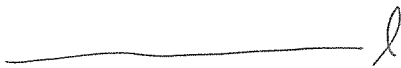
Conclude



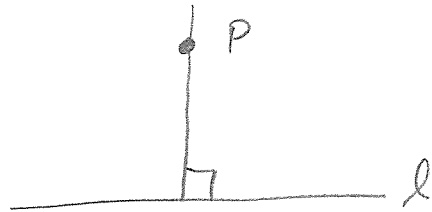
- Thm • Through a point outside a line, there is exactly one line perpendicular to the given line.

Given

$P \cdot$



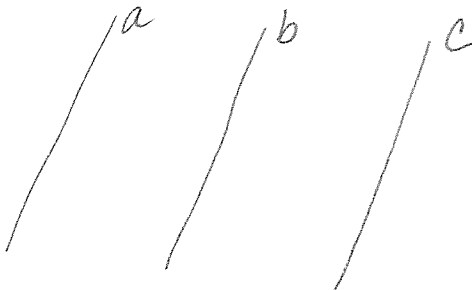
Conclude



- Thm • Two lines parallel to a third line are parallel to each other.

Given

$a \parallel b, c \parallel b$



Conclude

$a \parallel c$

Homework:

Text: p. 163/ Read "Constructing Parallel Lines" section.
Do Example A and B on this page. Do the best you can.