

The Perpendicular Bisector

You must be able to write the equation (find) of the perpendicular bisector to a line segment. Below is an example. Study it carefully and complete the problems.

Example: Find the perpendicular bisector of \overline{AB} given the following coordinates:

A (1, 7) B (13, 9)

Step 1: Find the midpoint of \overline{AB} . (Why? Since the perpendicular bisector intersects the segment at its midpoint, we have to know the midpoint.) **Important!** This is the point you must use in step 3 when you are looking for the y-intercept.

$$x_m = \frac{1+13}{2} \quad y_m = \frac{7+9}{2}$$

$$x_m = 7 \quad y_m = 8$$

midpoint (7, 8)

Step 2: Find the slope of \overline{AB} . (Why? Since the perpendicular bisector is *perpendicular* to the segment, we can find the slope of it by finding the slope of \overline{AB} and using its negative reciprocal.

$$m_{\overline{AB}} = \frac{9-7}{13-1} \rightarrow \frac{2}{12} = \frac{1}{6} \rightarrow m_{\perp} = -6$$

Step 3: Find the y-intercept. Remember to use the midpoint from step 1 and the m_{\perp} from step 2.

(7, 8) $y = mx + b$
 $\uparrow \uparrow$ $8 = -6(7) + b$
 x y $b = 50$

$y = -6x + 50$

This is the perpendicular bisector _____

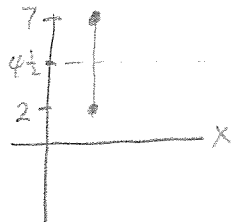
Find the perpendicular bisector for each line segment below.

1. \overline{AB} if A (2,2) and B (2, 7)

① $x_m = \frac{2+2}{2} = 2$ $y_m = \frac{2+7}{2} = \frac{9}{2}$

② $m = \frac{7-2}{2-2} = \text{undefined} \rightarrow \text{vertical line}$
 $m_{\perp} = 0 \rightarrow \text{horizontal line}$

Midpt (2, $\frac{9}{2}$)



③ $y = \frac{9}{2}$

2. \overline{AB} if A (-4, -3) and B (6, 2)

$$\begin{aligned}
 X_m &= \frac{-4+6}{2} & Y_m &= \frac{-3+2}{2} \\
 &= 1 & &= -\frac{1}{2} \\
 \text{midpt} & \left(1, -\frac{1}{2} \right)
 \end{aligned}
 \left\{ \begin{aligned}
 m_{\overline{AB}} &= \frac{2+3}{6+4} = \frac{5}{10} = \frac{1}{2} \\
 m_{\perp} &= -2 \\
 -\frac{1}{2} &= -2(1) + b \\
 \frac{1}{2} &= b
 \end{aligned} \right.$$

$$y = -2x + \frac{3}{2}$$

3. \overline{AB} if A (-6, -4) and B (1, -4)

$$\begin{aligned}
 X_m &= \frac{-6+1}{2} & Y_m &= \frac{-4-4}{2} \\
 &= -\frac{5}{2} & &= -4 \\
 \text{midpt} & \left(-\frac{5}{2}, -4 \right)
 \end{aligned}
 \left\{ \begin{aligned}
 m_{\overline{AB}} &= \frac{-4+4}{-6-1} = \emptyset \\
 m_{\perp} &= \text{undefined} \rightarrow \text{vertical line}
 \end{aligned} \right.$$

$$x = -\frac{5}{2}$$

4. \overline{AB} if A (-7, -2) and B (-1, -7)

$$\begin{aligned}
 X_m &= \frac{-7-1}{2} & Y_m &= \frac{-2-7}{2} \\
 &= -4 & &= -\frac{9}{2} \\
 \text{midpt} & \left(-4, -\frac{9}{2} \right)
 \end{aligned}
 \left\{ \begin{aligned}
 m_{\overline{AB}} &= \frac{-2+7}{-7+1} = \frac{5}{-6} \\
 m_{\perp} &= \frac{6}{5} \\
 -\frac{9}{2} &= \frac{6}{5}(-4) + b \\
 -\frac{45}{10} + \frac{48}{10} &= b
 \end{aligned} \right.$$

$$y = \frac{6}{5}x + \frac{3}{10}$$

5. \overline{AB} if A (-8, 4) and B (-4, 7)

$$\begin{aligned}
 X_m &= \frac{-8-4}{2} & Y_m &= \frac{4+7}{2} \\
 &= -6 & &= \frac{11}{2} \\
 \text{midpt} & \left(-6, \frac{11}{2} \right)
 \end{aligned}
 \left\{ \begin{aligned}
 m_{\overline{AB}} &= \frac{7-4}{-4+8} = \frac{3}{4} \\
 m_{\perp} &= -\frac{4}{3} \\
 \frac{11}{2} &= -\frac{4}{3}(-6) + b \\
 5\frac{1}{2} - 8 &= b \\
 -2\frac{1}{2} &= b
 \end{aligned} \right.$$

$$y = -\frac{4}{3}x - \frac{5}{2}$$

Geometry (H)

Homework – Section 13.1, 13.2, 13.3, 13.5

Name: _____

KEY

For problems #1 – 10, use the following points.

A(3,8), B(-2,0), C(1,-4), D(-6,7), E(4,-4), F(3,-2)

1. Find the slope of \overline{DB} .

$$m = \frac{7-0}{-6-2} = \frac{7}{-4}$$

2. Find the slope of \overline{AF} .

$$m = \frac{8-(-2)}{3-3} = \text{undefined}$$

3. Find CD. $d = \sqrt{(1-(-6))^2 + (-4-7)^2}$

$$= \sqrt{49 + 121}$$

$$CD = \sqrt{170}$$

4. Find EF.

$$EF = \sqrt{(4-3)^2 + (-4-(-2))^2}$$

$$= \sqrt{1+4}$$

$$EF = \sqrt{5}$$

5. Find the midpoint of \overline{AC} .

$$x = \frac{3+1}{2} \quad y = \frac{8+(-4)}{2}$$

$$(2, 2)$$

6. Find the midpoint of \overline{BD} . $(-4, 3\frac{1}{2})$

$$x = \frac{-2+(-6)}{2} \quad y = \frac{0+7}{2}$$

$$= -4 \quad = 3\frac{1}{2}$$

7. Give the slope of the line

a. parallel to \overline{CD} . $m_{CD} = \frac{7+4}{-6-1} = \frac{11}{-7}$

b. perpendicular to \overline{CD} .

$$m = \frac{7}{11}$$

8. Find n so that the line containing A(2,-7) and B(n,9) has slope 5.

$$5 = \frac{9-(-7)}{n-2} \quad 5n - 10 = 16$$

$$5n = 26$$

$$n = 5\frac{1}{5}$$

9. What is the slope of the line $y = 3$?

$$m = 0$$

10. What is the slope of the line $x = -5$?

undefined

Use the distance, slope, and midpoint formulas to solve the following problems.

1. In an isosceles triangle, at least two sides of a triangle are congruent. Show that the following triangle is isosceles.

A(-3,4), M(3,1), Y(0,-2)

$$d_{AM} = \sqrt{(-3-3)^2 + (4-1)^2}$$

$$= \sqrt{36+9}$$

$$= \sqrt{45} = 3\sqrt{5}$$

$$d_{MY} = \sqrt{(3)^2 + (1+2)^2}$$

$$= \sqrt{9+9}$$

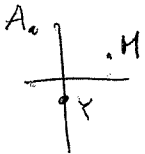
$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$d_{AY} = \sqrt{(-3)^2 + (4+2)^2}$$

$$= \sqrt{9+36}$$

$$= \sqrt{45} = 3\sqrt{5}$$



Since $AM = AY$, $\triangle AMY$ is an isosceles \triangle .

2. Find n so that the line containing A(2,-7) and B(n,9) has a slope of 5.

$$5 = \frac{9 - (-7)}{n - 2}$$

$$5n - 10 = 16$$

$$5n = 26$$

$$n = \frac{26}{5}$$

3. M is the midpoint of \overline{AB} . Find the coordinate of B using the midpoint formula.

M(2,1) A(-5,-3)

$$2 = \frac{-5+x}{2}$$

$$4 = -5+x$$

$$9 = x$$

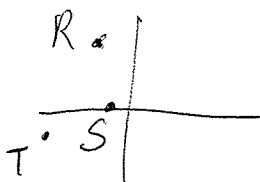
$$1 = \frac{-3+y}{2}$$

$$2 = -3+y$$

$$5 = y$$

$B(9, 5)$

4. R(-2,3), S(-1,0) and T(-4,-1). Show $\angle S$ is right using slope.



$$m_{RS} = \frac{0-3}{-1-(-2)} = \frac{-3}{1} = -3$$

$$m_{TS} = \frac{-1-0}{-4-(-1)} = \frac{-1}{-3} = \frac{1}{3}$$

$$m_{TR} = \frac{3-(-1)}{-2-(-4)} = \frac{4}{2} = 2$$

Since m_{RS} & m_{TS} are negative reciprocals, $\overline{RS} \perp \overline{TS}$. Therefore, \overline{RS} & \overline{TS} form a rt \angle .

