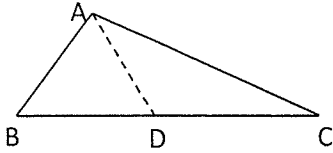


The Median

You must be able to write the equation (find) of the median to one side of a triangle. What is a median? Definition: A median is the line segment that connects the vertex of a triangle to the midpoint of the side opposite from that vertex.



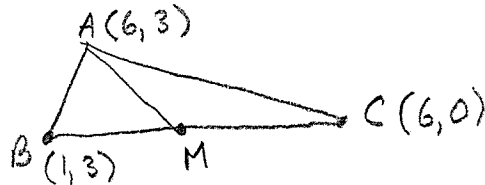
If D is the midpoint of \overline{BC} , then \overline{AD} is the median to side \overline{BC} .

Below is an example. Study it carefully and complete the problems that follow.

Example: Find the median to \overline{BC} in ΔABC given the following coordinates:

A (6, 3) B (1, 3) C (6, 0)

Step 1: Understand how the median is situated in ΔABC . Let's call the median \overline{AM} . "Median to \overline{BC} " means the midpoint of \overline{BC} will be one endpoint of the median. The other endpoint must be vertex A of the triangle.



Step 2: Find the midpoint "M" of \overline{BC} . (Why? This point will be one of the two points you will use to find the slope of the median.)

$$\begin{aligned} x_M &= \frac{1+6}{2} & y_M &= \frac{3+0}{2} \\ &= \frac{7}{2} & &= \frac{3}{2} \end{aligned} \quad \text{midpoint } M \left(3\frac{1}{2}, 1\frac{1}{2} \right)$$

Step 3: Find the slope of the median. (Why? You'll need it to find the y-intercept and to write the equation.)

M $\left(\frac{7}{2}, \frac{3}{2} \right)$ A (6, 3)

$$m = \frac{3 - \frac{3}{2}}{6 - \frac{7}{2}} = \frac{\frac{3}{2}}{2\frac{1}{2}} \rightarrow \frac{3}{2} \div \frac{5}{2} = \frac{3}{2} \cdot \frac{2}{5} = \frac{3}{5} \quad m = \frac{3}{5}$$

Step 4: Find the y-intercept of \overline{AM} . You can use either Point A or point M.

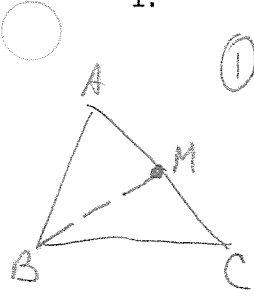
$$A(6, 3) \quad \text{slope} = \frac{3}{5} \quad \begin{aligned} y &= mx + b \\ 3 &= \frac{3}{5}(6) + b \rightarrow b = -\frac{3}{5} \end{aligned}$$

This is the median

$y = \frac{3}{5}x - \frac{3}{5}$

Find the median to the indicated side in $\triangle ABC$ below. Draw a diagram for each problem.

1. Median \overline{BM} to side \overline{AC} if $A(-3, -3)$ $B(3, 2)$ $C(5, -3)$



① midpoint of \overline{AC} $\left(\frac{-3+5}{2}, \frac{-3-3}{2}\right)$ ③ $2 = \frac{5}{2}(3) + b$

$M = (1, -3)$

$\frac{4}{2} - \frac{15}{2} = b$

$b = -\frac{11}{2}$

② $m_{\overline{BM}} = \frac{-3-2}{1-3}$
 $= \frac{-5}{-2} = \frac{5}{2}$

④ $y = \frac{5}{2}x - \frac{11}{2}$

2. Median \overline{CM} to side \overline{AB} if $A(0, 3)$ $B(0, -4)$ $C(7, 3)$



① midpoint M of \overline{AB} ② $m_{\overline{CM}} = \frac{3 - (-4)}{7 - 0}$

$\left(\frac{0+0}{2}, \frac{3-4}{2}\right)$

$= \frac{7}{7} = 1$

$M(0, -\frac{1}{2})$

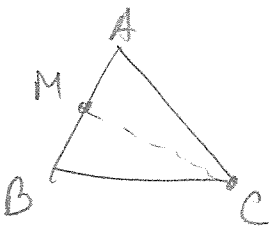
③ $3 = \frac{1}{2}(7) + b$

$\frac{6}{2} - \frac{7}{2} = b$

$-\frac{1}{2} = b$

$y = \frac{1}{2}x - \frac{1}{2}$

3. Median \overline{CM} to side \overline{AB} if $A(-3, 0)$ $B(3, 6)$ $C(2, -6)$



① $M\left(\frac{-3+3}{2}, \frac{0+6}{2}\right)$ ② $m_{\overline{CM}} = \frac{-6-3}{2-0} = -\frac{9}{2}$

$(0, 3)$

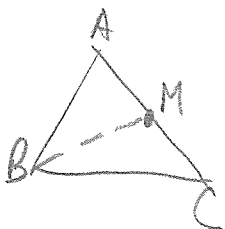
③ $-6 = -\frac{9}{2}(2) + b$

$-6 + 9 = b$

$3 = b$

$y = -\frac{9}{2}x + 3$

4. Median \overline{BM} to side \overline{AC} if $A(-8, 6)$ $B(-3, -3)$ $C(0, 3)$



① $M\left(\frac{-8+0}{2}, \frac{6+3}{2}\right)$ ② $m_{\overline{BM}} = \frac{-3-4\frac{1}{2}}{-3+4} = \frac{-7\frac{1}{2}}{1} = -\frac{15}{2}$

$\left(-4, \frac{9}{2}\right)$

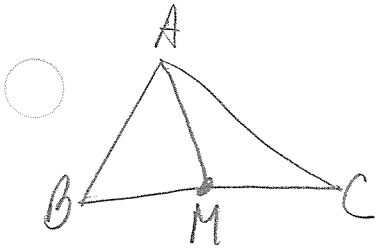
③ $-3 = -\frac{15}{2}(-3) + b$

$-\frac{6}{2} - \frac{45}{2} = b$

$-\frac{51}{2} = b$

$y = -\frac{15}{2}x - \frac{51}{2}$

5. Median \overline{AM} to side \overline{BC} if $A(-4, -4)$ $B(-16, 2)$ $C(-4, -12)$



$$\textcircled{1} M \left(\frac{-16-4}{2}, \frac{2-12}{2} \right)$$

of \overline{BC}

$$M(-10, -5)$$

$$\textcircled{2} m_{\overline{AM}} = \frac{-4+5}{-4+10} = \frac{1}{6}$$

$$\textcircled{3} -4 = \frac{1}{6}(-4) + b$$

$$\frac{-24}{6} + \frac{4}{6} = b$$

$$\frac{-20}{6} = \frac{-20}{6} = b$$

$$y = \frac{1}{6}x - \frac{10}{3}$$

Practice I

1. Show that $\triangle EBD$ is an isosceles triangle. $E(-5, 2)$, $B(-2, -2)$, and $D(3, -2)$.

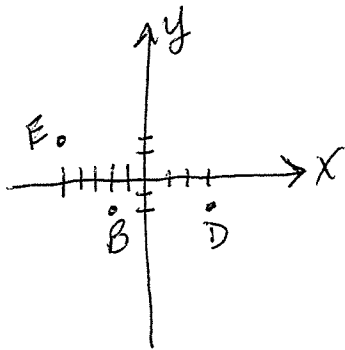
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2. Use slope to show that $\triangle AEB$ is **NOT** a right triangle. $A(0, 5)$, $E(-5, 2)$, $B(-2, -2)$.

3. Find the slope of the median of $\triangle ABC$ drawn from point B. $A(1, 4)$, $B(-1, -3)$, $C(4, -5)$.

4. M is the midpoint of \overline{AB} , where the coordinates of A are given. Find the coordinates of B. $A(-1, -2)$ and $M(5, 1)$

11. Show that $\triangle EBD$ is an isosceles triangle. $E(-5,2)$, $B(-2,-2)$ and $D(3,-2)$.



$$d_{EB} = \frac{\sqrt{(-5 - (-2))^2 + (2 - (-2))^2}}{\sqrt{9 + 16}} = \frac{\sqrt{9 + 16}}{\sqrt{25}}$$

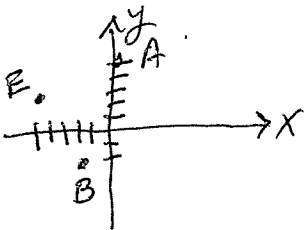
$$d_{BD} = \frac{\sqrt{(3 - (-2))^2 + (-2 - (-2))^2}}{\sqrt{25}}$$

$$d_{EB} = 5$$

$$d_{BD} = 5$$

Since $BE = BD$, $\triangle EBD$ is isosceles.

12. Use slope to show that $\triangle AEB$ is **NOT** a right triangle. $A(0,5)$, $E(-5,2)$, $B(-2,-2)$.



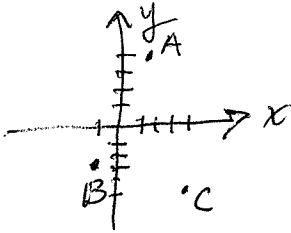
$$m_{EA} = \frac{5 - 2}{0 - (-5)} = \frac{3}{5}$$

$$m_{AB} = \frac{5 - (-2)}{0 - (-2)} = \frac{7}{2}$$

$$m_{EB} = \frac{2 - (-2)}{-5 - (-2)} = \frac{4}{-3}$$

none of the slopes are negative reciprocals.
Therefore, $\triangle AEB$ is NOT a Rt. \triangle .

13. Find the slope of the altitude of $\triangle ABC$ drawn from point A. $A(1,4)$, $B(-1,-3)$, $C(4,-5)$.

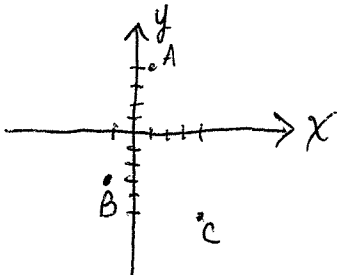


$$m_{AB} = \frac{-3 - 4}{-1 - 1} = \frac{-7}{-2} = \frac{7}{2}$$

$$m_{BC} = \frac{-5 - (-3)}{4 - (-1)} = \frac{-2}{5}$$

Slope of altitude = $\frac{5}{2}$

14. Find the slope of the median of $\triangle ABC$ drawn from point B. $A(1,4)$, $B(-1,-3)$, $C(4,-5)$.



$$\text{midpt of } \overline{AC} = (2.5, -0.5)$$

$$x = \frac{1 + 4}{2} = 2.5$$

$$y = \frac{4 + (-5)}{2} = -0.5$$

Slope of median:

$$m = \frac{-3 - (-0.5)}{-1 - 2.5} = \frac{-2.5}{-3.5} = \frac{5}{7}$$

$$= \frac{-2.5}{-3.5} = \frac{5}{7} = \left(\frac{5}{7}\right)$$

15. M is the midpoint of \overline{AB} , where the coordinates of A are given. Find the coordinates of B.

$$x_m = \frac{x_A + x_B}{2}$$

$A(-1, -2)$ and $M(5, 1)$

$$5 = \frac{-1 + x}{2}$$

$$1 = \frac{-2 + y}{2}$$

$$10 = -1 + x$$

$$2 = -2 + y$$

$$11 = x$$

$$4 = y$$

$$B = (11, 4)$$

PRACTICE 2

Use the distance, slope and midpoint formulas to answer the following problems.

1. The distance between (1,2) and (x,8) is 10. Find x.

$$x = -7 \quad x = 9$$

2. A, B and C are collinear. A(-4,-2), B(-1,1) and C(3,t). Find t, using slope.

$$t = 5$$

3. M is the midpoint of \overline{AB} . M(5,1), A(1,-3), find B.

$$B(9,5)$$

4. R(-2,3), S(-1,0) and T(-4,-1). Is $\triangle RST$ right?

$$m_{RS} = -3$$

$$m_{TS} = \frac{1}{3}$$

Yes