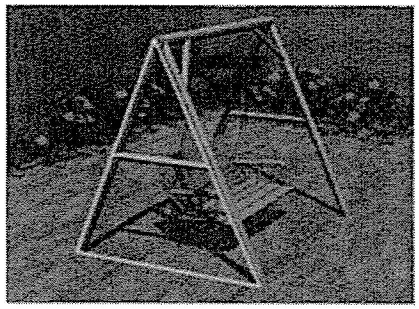
Geometry (H) date *Notes*
Chap: Special Parts of the Triangle

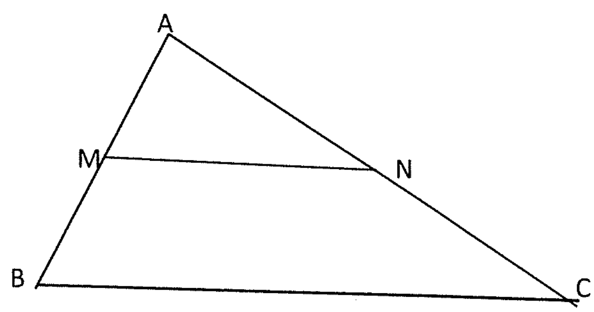
The Midsegment of a Triangle

The midsegment of a triangle is a line segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments. Midsegments are often used to add rigidity to structures. In the support for the garden swing shown, the crossbar $\overline{DE}$ is a midsegment of $\triangle ABC$



The Midsegment Theorem:

The segment that joins the midpoints of two sides of a triangle
(1) is parallel to the third side;
(2) is half as long as the third side.



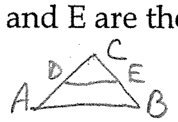
Given: M is the midpoint of $\overline{AB}$
N is the midpoint of $\overline{AC}$

You conclude: (1) $\overleftrightarrow{MN} \parallel \overline{BC}$

(2) $MN = \frac{1}{2} BC$

Ex 1: Verifying the Midsegment Theorem Algebraically:

ΔABC has vertices A(-7, -1), B(-5, 5), and C(1, 3). D and E are the midpoints of $\overline{AC}$, and $\overline{BC}$ respectively. Verify the midsegment theorem.



① Look for midpoints.

$D\left(\frac{-7+1}{2}, \frac{-1+3}{2}\right)$

$D(-3, 1)$

$E\left(\frac{-5+1}{2}, \frac{5+3}{2}\right)$

$E(-2, 4)$

② Verify $AB = 2DE$

$AB = \sqrt{(-7+5)^2 + (-1-5)^2} = \sqrt{40} = 2\sqrt{10}$
4 36

$DE = \sqrt{(-3+2)^2 + (1-4)^2} = \sqrt{10}$
1 + 9

It is verified that AB is twice DE.

③ Verify $\overline{DE} \parallel \overline{AB}$.

$m_{\overline{DE}} = \frac{4-1}{-2+3} = \frac{3}{1} =$

$m_{\overline{AB}} = \frac{-1-5}{-7+5} = \frac{-6}{-2} = 3$

$\overline{DE} \parallel \overline{AB}$ since they have same slopes.

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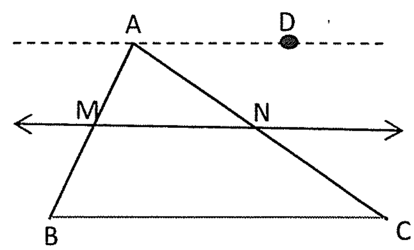
Two More Theorems

Theorem: A line that contains the midpoint of one side of a triangle and is parallel to another side passes through the midpoint of the third side.

Given: M is the midpoint of $\overline{AB}$

$\overleftrightarrow{MN} \parallel \overline{BC}$

You conclude: N is the midpoint of $\overline{AC}$



Theorem: If 2 lines are parallel, then all points on one line are equidistant from the points on the other line.

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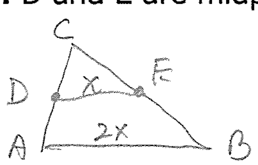
Ex 2: D and E are midpoints of $\overline{AC}$ and $\overline{BC}$, of ΔABC respectively. AB+DE=36. Find AB and DE.



$AB + DE = 36$

$2x + x = 36$

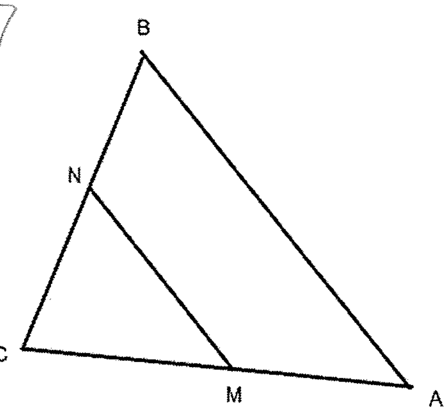
$x = 12$

$DE = 12$

Ex 3: $\overline{MN}$ is a midsegment of the given triangle. $AM = x + 5, MC = 2y + 6,$ $MN = 2x - 5, and\ AB = y + 8$. Find MN and AB.

$\boxed{\begin{array}{l} MN = 5 \\ AB = 10 \end{array}}$

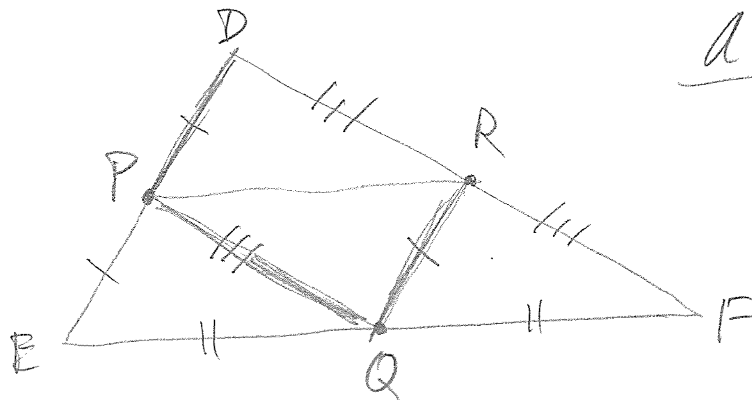
check if
$MN = \frac{1}{2}AB$



Ex4: Explore: P, Q, and R are midpoints of the sides of ΔDEF, respectively.

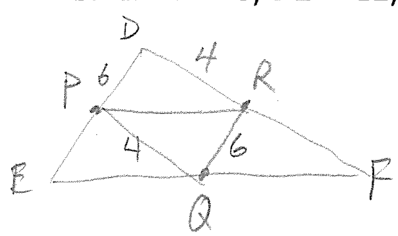
a. What kind of figure is DPQR?

a parallelogram

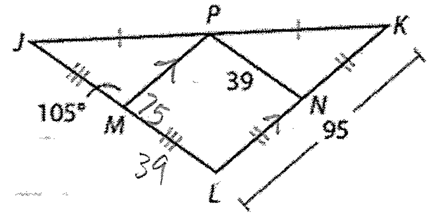


b. If DF = 8, DE = 12, and FE = 10, what is the perimeter of DPQR?

Perimeter = 20



EX5: Find JL, PM, and m∠MLK.

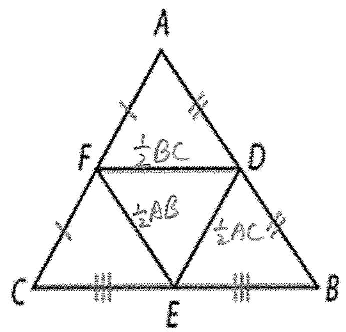


$JL = 2(39) = 78$

$PM = \frac{95}{2} = 47\frac{1}{2}$

$m\angle MLK = 105°$

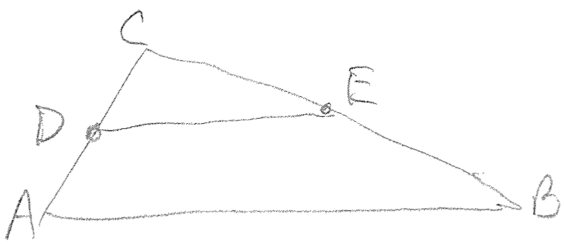
EX6: How does the perimeter of $\triangle$DEF compare to that of $\triangle$ABC? Explain.



$$P_{\triangle DEF} = \frac{1}{2}AB + \frac{1}{2}BC + \frac{1}{2}AC$$
$$\frac{1}{2}(AB + BC + AC)$$
$$P_{\triangle ABC} = AB + BC + AC$$

So, perimeter of $\triangle DEF$ is $\frac{1}{2}$ that of $\triangle ABC$.

Ex7: ΔABC has vertices A(-1, 6), B(-4, -3), and C(7, -5). D and E are the midpoints of $\overline{AC}$, and $\overline{BC}$ respectively. Verify the midsegment theorem.



① Midpts

$D\left(\frac{-1+7}{2}, \frac{6-5}{2}\right)$

$D\left(3, \frac{1}{2}\right)$

$E\left(\frac{-4+7}{2}, \frac{-3-5}{2}\right)$

$E\left(\frac{3}{2}, -4\right)$

② $2DE = AB$?

$DE = \sqrt{\left(3-\frac{3}{2}\right)^2 + \left(\frac{1}{2}+4\right)^2}$

$\sqrt{\frac{9}{4} + \frac{81}{4}}$

$\sqrt{\frac{90}{4}} = \frac{\sqrt{90}}{2} = \frac{3\sqrt{10}}{2}$

$AB = \sqrt{(-1+4)^2 + (6+3)^2}$

$\sqrt{9+81}$

$\sqrt{90} = 3\sqrt{10}$

$2\left(\frac{3\sqrt{10}}{2}\right) = 3\sqrt{10}$

$2DE = AB$ ✓

③ $m_{\overline{DE}} = \frac{\frac{1}{2}+4}{3-\frac{3}{2}}$

$= \frac{9}{2} \div \frac{3}{2}$

$= \frac{9}{2} \cdot \frac{2}{3} = 3$

$m_{\overline{AB}} = \frac{6+3}{-1+4} = \frac{9}{3} = 3$

Since slopes are same, $\overline{DE} \parallel \overline{AB}$.