Geometry Honors | Date:

Chapter: Special Concepts of the Triangle

8.2 Angle Bisectors of Triangles

Essential Question: How can you use angle bisectors to find the point that is equidistant from all the sides of a triangle?

Recall:

"Distance form a point to a line" – refers to the shortest distance from a point to a line;
-is the length of the perpendicular segment from the point to the line/segment;

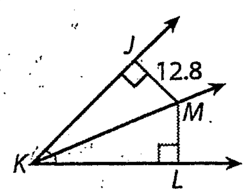
Recall two theorems we've studied:

1. If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.
   diagram:

2. If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.
   diagram:

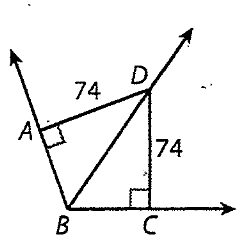
Example 1 **Find each measure.**

(A) LM



$\overrightarrow{KM}$ is the bisector of $\angle JKL$, so $LM = JM = 12.8$.

(B) $m\angle ABD$, given that $m\angle ABC = 112°$



Since $AD = DC$, $\overline{AD} \perp \overrightarrow{BA}$, and $\overline{DC} \perp \overrightarrow{BC}$, you know that $\overrightarrow{BD}$ bisects $\angle ABC$ by the ______________________ Theorem.

So, $m\angle ABD = \frac{1}{2} m\angle$ ______ $=$ ☐°.

Reflect

3. In the Converse of the Angle Bisector Theorem, why is it important to say that the point must be in the *interior* of the angle?

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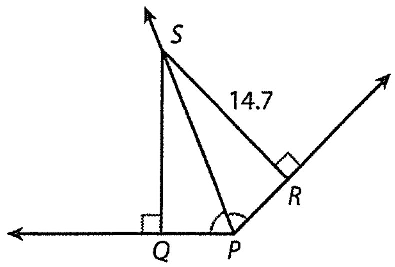
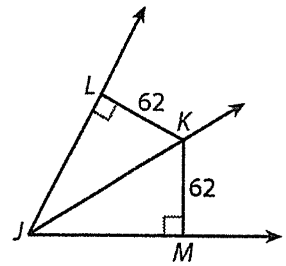
Your Turn

Find each measure.

4. *QS*

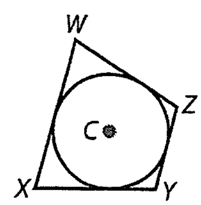


5. m∠*LJM*, given that $m\angle KJM = 29°$



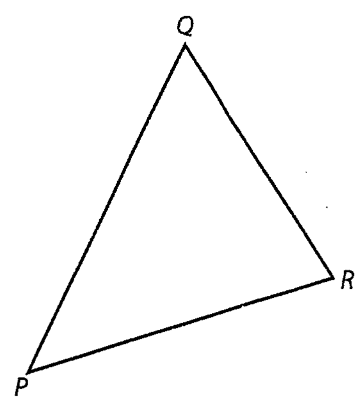
Explain 2 Constructing an Inscribed Circle

A circle is **inscribed** in a polygon if each side of the polygon is tangent to the circle. In the figure, circle *C* is inscribed in quadrilateral *WXYZ* and this circle is called the **incircle (inscribed circle)** of the quadrilateral.



In order to construct the incircle of a triangle, you need to find the center of the circle. This point is called the **incenter** of the triangle.

Example 2 **Use a compass and straightedge to construct the inscribed circle of $\triangle PQR$.**



Step 1 The center of the inscribed circle must be equidistant from $\overline{PQ}$ and $\overline{PR}$. What is the set of points equidistant from $\overline{PQ}$ and $\overline{PR}$? ____________________
Construct this set of points.

Step 2 The center must also be equidistant from $\overline{PR}$ and $\overline{QR}$. What is the set of points equidistant from $\overline{PR}$ and $\overline{QR}$?
____________________ Construct this set of points.

Step 3 The center must lie at the intersection of the two sets of points you constructed. Label this point *C*.

Step 4 Place the point of your compass at *C* and open the compass until the pencil just touches a side of $\triangle PQR$. Then draw the inscribed circle.

Reflect

6. Suppose you started by constructing the set of points equidistant from $\overline{PR}$ and $\overline{QR}$, and then constructed the set of points equidistant from $\overline{QR}$ and $\overline{QP}$. Would you have found the same center point? Check by doing this construction.

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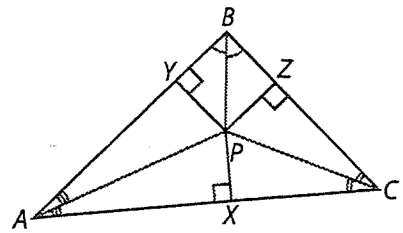
Explain 3 Using Properties of Angle Bisectors

As you have seen, the angle bisectors of a triangle are concurrent. The point of concurrency is the incenter of the triangle.

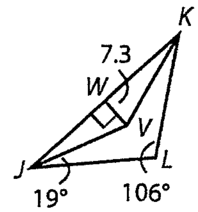
Incenter Theorem

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

$PX = PY = PZ$



Example 3 $\overline{JV}$ **and** $\overline{KV}$ **are angle bisectors of** $\triangle JKL$**. Find each measure.**



(A) the distance from V to $\overline{KL}$

V is the incenter of $\triangle JKL$. By the Incenter Theorem, V is equidistant from the sides of $\triangle JKL$. The distance from V to $\overline{JK}$ is 7.3. So the distance from V to $\overline{KL}$ is also 7.3.

(B) $m\angle VKL$

$\overline{JV}$ is the bisector of $\angle$ ☐. $m\angle KJL = 2(\square) = \square$

Triangle Sum Theorem $\square + \square + m\angle JKL = 180°$

Subtract ☐ from each side. $m\angle JKL = \square$

$\overline{KV}$ is the bisector of $\angle JKL$. $m\angle VKL = \frac{1}{2}(\square) = \square$

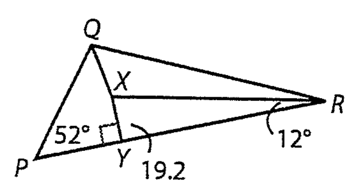
Reflect

7. In Part A, is there another distance you can determine? Explain.

Your Turn

$\overline{QX}$ **and** $\overline{RX}$ **are angle bisectors of** $\triangle PQR$**. Find each measure.**

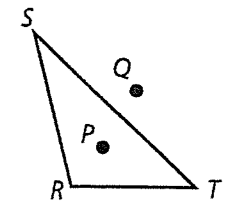
8. the distance from X to $\overline{PQ}$



9. $m\angle PQX$

Elaborate

10. *P* and *Q* are the circumcenter and incenter of $\triangle RST$, but not necessarily in that order. Which point is the circumcenter? Which point is the incenter? Explain how you can tell without constructing any bisectors.



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11. Complete the table by filling in the blanks to make each statement true.

	Circumcenter	Incenter
Definition	The point of concurrency of the ____________	The point of concurrency of the ____________
Distance	Equidistant from the ____________	Equidistant from the ____________
Location (Inside, Outside, On)	Can be ____________ the triangle	Always ________ the triangle

12. **Essential Question Check-In** How do you know that the intersection of the bisectors of the angles of a triangle is equidistant from the sides of the triangle?

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Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Use a compass and straightedge to investigate points on the bisector of an angle. On a separate piece of paper, draw a large angle *A*.

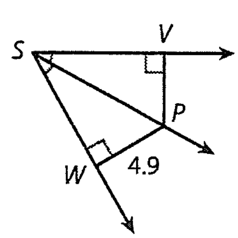
a. Construct the bisector of $\angle A$.

b. Choose a point on the angle bisector you constructed. Label it *P*. Construct a perpendicular through *P* to each side of $\angle A$.

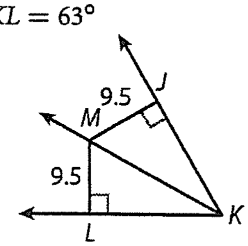
c. Explain how to use a compass to show that *P* is equidistant from the sides of $\angle A$.

Find each measure.

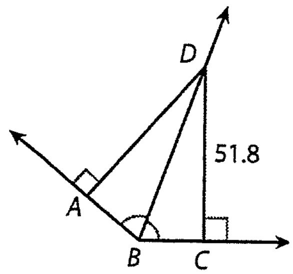
2. *VP*



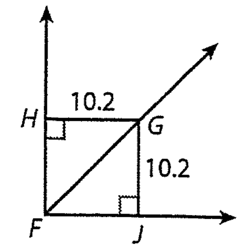
3. $m\angle LKM$, given that $m\angle JKL = 63°$



4. AD

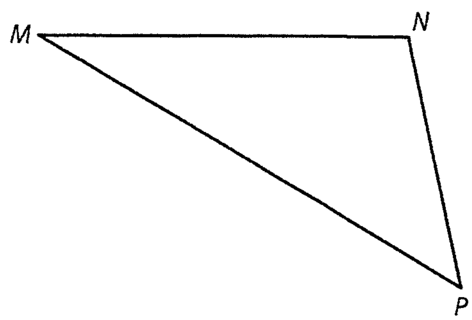


5. $m\angle HFJ$, given that $m\angle GFJ = 45°$

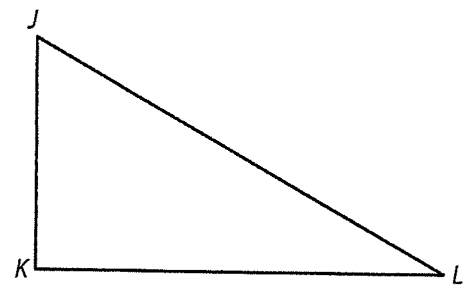


Construct an inscribed circle for each triangle.

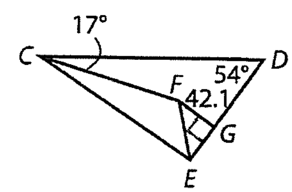
6.



7.



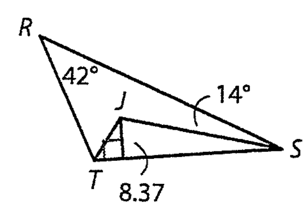
$\overline{CF}$ and $\overline{EF}$ are angle bisectors of $\triangle CDE$. Find each measure.



8. the distance from F to $\overline{CD}$

9. $m\angle FED$

$\overline{TJ}$ and $\overline{SJ}$ are angle bisectors of $\triangle RST$. Find each measure.



10. the distance from J to $\overline{RS}$

11. $m\angle RTJ$