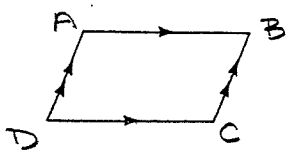
Geometry (H)
Section Notes – Parallelograms

Directions: *Read the following notes about parallelograms and their properties. Then use the notes to complete the attached problems.*

Definition: A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

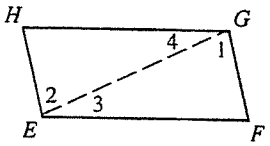


▱ABCD

Theorem **Opposite sides of a parallelogram are congruent.**

Given: ▱ EFGH

Prove: $\overline{EF} \cong \overline{HG}$; $\overline{FG} \cong \overline{EH}$



Proof:

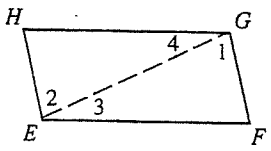
① Draw $\overline{EG}$.
② ▱ EFGH → ③ $\overline{HG} \parallel \overline{EF}$ → ④ $\angle 4 \cong \angle 3$
→ $\overline{HE} \parallel \overline{GF}$ → $\angle 2 \cong \angle 1$
⑤ $\overline{EG} \cong \overline{EG}$
→ ⑥ $\triangle HEG \cong \triangle FGE$ → ⑦ $\overline{EF} \cong \overline{HG}$
→ $\overline{FG} \cong \overline{EH}$

① Two points determine a line.
② Given
③ ▱ → opp sides parallel.
④ 2 ∥ lines → alt. int. ∠s ≅.
⑤ Reflexive Prop.
⑥ ASA post.
⑦ CPCTC

Theorem **Opposite angles of a parallelogram are congruent.**

Given: ▱ EFGH

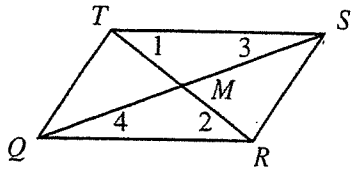
Prove: $\angle H \cong \angle F$ and $\angle E \cong \angle G$



Theorem **Diagonals of a parallelogram bisect each other.**

Given: □QRST with diagonals $\overline{QS}$ and $\overline{TR}$

Prove: $\overline{QS}$ and $\overline{TR}$ bisect each other



Show $\triangle TMS \cong \triangle RMQ$

① □QRST → ② $\overline{TS} \parallel \overline{QR}$ → ③ $\angle 3 \cong \angle 4$, $\angle 1 \cong \angle 2$ → ⑤ $\triangle TMS \cong \triangle RMQ$

② $\overline{TS} \parallel \overline{QR}$ → ④ $\overline{TS} \cong \overline{QR}$ → ⑤ $\triangle TMS \cong \triangle RMQ$

⑤ $\triangle TMS \cong \triangle RMQ$ → ⑥ $\overline{TM} \cong \overline{MR}$, $\overline{QM} \cong \overline{MS}$ → ⑦ M is a midpt. → ⑧ $\overline{QS}$ & $\overline{TR}$ bisect each other.

① Given

② □ → opp sides parallel

③ 2 ∥ lines → alt. int. $\angle$s $\cong$.

④ □ → opp sides $\cong$.

⑤ AAS $\cong$ post.

⑥ CPCTC

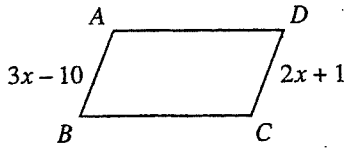
⑦ If a pt divides a seg. into 2 $\cong$ parts, it's a midpt.

⑧ A segment that goes thru a midpt is a seg. bisector

Corollary: Consecutive angles in a parallelogram are supplementary.

Example 1

Find the lengths of $\overline{AB}$ and $\overline{DC}$ in □$ABCD$.



Solution

$3x - 10 = 2x + 1$ *Opposite sides of a □ have equal measure.*

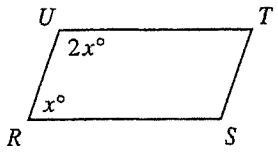
$x = 11$

$2(11) + 1 = 23$ *Replace x with 11 in 2x + 1.*

$AB = DC = 23$

Example 2

Find the measures of the four angles of □$RSTU$ if the measure of $\angle U$ is twice the measure of $\angle R$.



Solution

$x + 2x = 180$ *Consecutive angles of a □ are supplementary.*

$3x = 180$

$x = 60$

$m\angle R = m\angle T = 60$ $m\angle U = m\angle S = 120$