

# Review Exercises

## Exponential and Logarithmic Functions

Blue problem numbers indicate the authors' suggestions for use in a Practice Test.

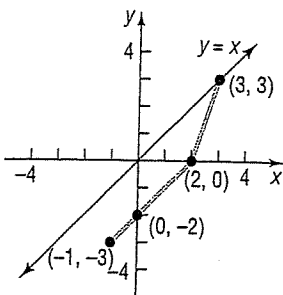
In Problems 1 and 2, (a) find the inverse of the function, and (b) determine whether the inverse represents a function.

1.  $\{(1, 2), (3, 5), (5, 8), (6, 10)\}$

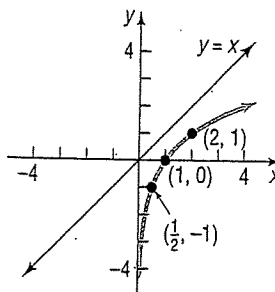
2.  $\{(-1, 4), (0, 2), (1, 4), (3, 7)\}$

In Problems 3 and 4, the graph of a one-to-one function is given. Draw the graph of the inverse function  $f^{-1}$ . For convenience (and as a hint), the graph of  $y = x$  is also given.

3.



4.



In Problems 5–10, the function  $f$  is one-to-one. Find the inverse of each function and check your answer. Find the domain and range of  $f$  and  $f^{-1}$ . Use a graphing utility to simultaneously graph  $f$ ,  $f^{-1}$ , and  $y = x$  on the same square screen.

5.  $f(x) = \frac{2x + 3}{5x - 2}$

6.  $f(x) = \frac{2 - x}{3 + x}$

7.  $f(x) = \frac{1}{x - 1}$

8.  $f(x) = \sqrt{x - 2}$

9.  $f(x) = \frac{3}{x^{1/3}}$

10.  $f(x) = x^{1/3} + 1$

In Problems 11 and 12, suppose that  $f(x) = 3^x$  and  $g(x) = \log_3 x$ .

11. Evaluate the following: (a)  $f(4)$  (b)  $g(9)$  (c)  $f(-2)$  (d)  $g\left(\frac{1}{27}\right)$

12. Evaluate the following: (a)  $f(1)$  (b)  $g(81)$  (c)  $f(-4)$  (d)  $g\left(\frac{1}{243}\right)$

In Problems 13 and 14, convert each exponential expression to an equivalent expression involving a logarithm. In Problems 15 and 16, convert each logarithmic expression to an equivalent expression involving an exponent.

13.  $5^2 = z$

14.  $a^5 = m$

15.  $\log_5 u = 13$

16.  $\log_a 4 = 3$

In Problems 17–20, find the domain of each logarithmic function.

17.  $\log(3x - 2)$

18.  $\log_5(2x + 1)$

19.  $\log_2(x^2 - 3x + 2)$

20.  $\ln(x^2 - 9)$

In Problems 21–26, evaluate each expression. Do not use a graphing utility.

21.  $\log_2\left(\frac{1}{8}\right)$

22.  $\log_3 81$

23.  $\ln e^{\sqrt{2}}$

24.  $e^{\ln 0.1}$

25.  $2^{\log_2 0.4}$

26.  $\log_2 2^{\sqrt{3}}$

In Problems 27–32, write each expression as the sum and/or difference of logarithms. Express powers as factors.

27.  $\log_3\left(\frac{uv^2}{w}\right)$

28.  $\log_2(a^2\sqrt{b})^4$

29.  $\log(x^2\sqrt{x^3 + 1})$

30.  $\log_5\left(\frac{x^2 + 2x + 1}{x^2}\right)$

31.  $\ln\left(\frac{x\sqrt{x^2 + 1}}{x - 3}\right)$

32.  $\ln\left(\frac{2x + 3}{x^2 - 3x + 2}\right)^2$

In Problems 33–38, write each expression as a single logarithm.

33.  $3 \log_4 x^2 + \frac{1}{2} \log_4 \sqrt{x}$

34.  $-2 \log_3\left(\frac{1}{x}\right) + \frac{1}{3} \log_3 \sqrt{x}$

35.  $\ln\left(\frac{x-1}{x}\right) + \ln\left(\frac{x}{x+1}\right) - \ln(x^2 - 1)$

36.  $\log(x^2 - 9) - \log(x^2 + 7x + 12)$

37.  $2 \log 2 + 3 \log x - \frac{1}{2} [\log(x + 3) + \log(x - 2)]$

38.  $\frac{1}{2} \ln(x^2 + 1) - 4 \ln \frac{1}{2} - \frac{1}{2} [\ln(x - 4) + \ln x]$

In Problems 39 and 40, use the Change-of-Base Formula and a calculator to evaluate each logarithm. Round your answer to three decimal places.

39.  $\log_4 19$

40.  $\log_2 21$

In Problems 41 and 42, graph each function using a graphing utility and the Change-of-Base Formula.

41.  $y = \log_3 x$

42.  $y = \log_7 x$

In Problems 43–52, use transformations to graph each function. Determine the domain, range, and any asymptotes. Verify your results using a graphing utility.

43.  $f(x) = 2^{x-3}$

44.  $f(x) = -2^x + 3$

45.  $f(x) = \frac{1}{2}(3^{-x})$

46.  $f(x) = 1 + 3^{2x}$

47.  $f(x) = 1 - e^x$

48.  $f(x) = 3 + \ln x$

49.  $f(x) = 3e^x$

50.  $f(x) = \frac{1}{2} \ln x$

51.  $f(x) = 3 - e^{-x}$

52.  $f(x) = 4 - \ln(-x)$

In Problems 53–72, solve each equation. Verify your result using a graphing utility.

53.  $4^{1-2x} = 2$

54.  $8^{6+3x} = 4$

55.  $3^{x^2+x} = \sqrt{3}$

56.  $4^{x-x^2} = \frac{1}{2}$

57.  $\log_x 64 = -3$

58.  $\log_{\sqrt{2}} x = -6$

59.  $5^x = 3^{x+2}$

60.  $5^{x+2} = 7^{x-2}$

61.  $9^{2x} = 27^{3x-4}$

62.  $25^{2x} = 5^{x^2-12}$

63.  $\log_3 \sqrt{x-2} = 2$

64.  $2^{x+1} \cdot 8^{-x} = 4$

65.  $8 = 4^{x^2} \cdot 2^{5x}$

66.  $2^x \cdot 5 = 10^x$

67.  $\log_6(x+3) + \log_6(x+4) = 1$

68.  $\log_{10}(7x-12) = 2 \log_{10} x$

69.  $e^{1-x} = 5$

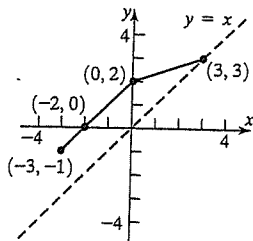
70.  $e^{1-2x} = 4$

71.  $2^{3x} = 3^{2x+1}$

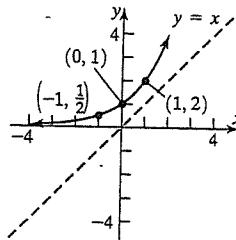
72.  $2^{x^3} = 3^{x^2}$

### Review Exercises

1. (a)  $\{(2, 1), (5, 3), (8, 5), (10, 6)\}$  (b) Inverse is a function



2. (a)  $\{(4, -1), (2, 0), (4, 1), (7, 3)\}$  (b) Inverse is not a function



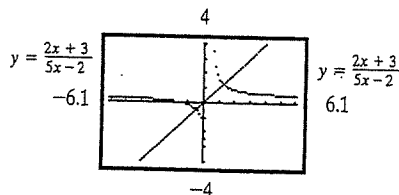
5.  $f^{-1}(x) = \frac{2x+3}{5x-2}$

$$f(f^{-1}(x)) = \frac{2\left(\frac{2x+3}{5x-2}\right) + 3}{5\left(\frac{2x+3}{5x-2}\right) - 2} = x$$

$$f^{-1}(f(x)) = \frac{2\left(\frac{2x+3}{5x-2}\right) + 3}{5\left(\frac{2x+3}{5x-2}\right) - 2} = x$$

Domain  $f$  = Range  $f^{-1}$  = all real numbers except  $\frac{2}{5}$

Range  $f$  = Domain  $f^{-1}$  = all real numbers except  $\frac{2}{5}$



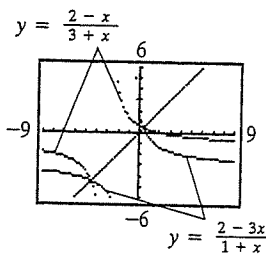
6.  $f^{-1}(x) = \frac{2-3x}{1+x}$

$$f(f^{-1}(x)) = \frac{2 - \frac{2-3x}{1+x}}{3 + \frac{2-3x}{1+x}} = x$$

$$f^{-1}(f(x)) = \frac{2 - 3\left(\frac{2-x}{3+x}\right)}{1 + \left(\frac{2-x}{3+x}\right)} = x$$

Domain  $f$  = Range  $f^{-1}$  = all real numbers except  $-3$

Range  $f$  = Domain  $f^{-1}$  = all real numbers except  $-1$

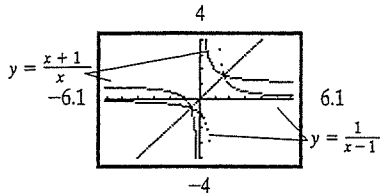


$$7. f^{-1}(x) = \frac{x+1}{x}$$

$$f(f^{-1}(x)) = \frac{1}{\frac{x+1}{x} - 1} = x$$

$$f^{-1}(f(x)) = \frac{\frac{1}{x} + 1}{\frac{1}{x} - 1} = x$$

Domain  $f$  = Range  $f^{-1}$  = all real numbers except 1  
 Range  $f$  = Domain  $f^{-1}$  = all real numbers except 0



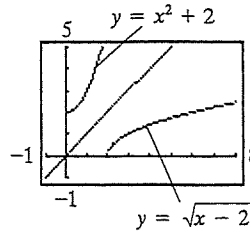
$$8. f^{-1}(x) = x^2 + 2, x > 0$$

$$f(f^{-1}(x)) = \sqrt{x^2 + 2} - 2 = x$$

$$f^{-1}(f(x)) = (\sqrt{x-2})^2 + 2 = x$$

Domain  $f$  = Range  $f^{-1}$  =  $(2, \infty)$

Range  $f$  = Domain  $f^{-1}$  =  $(0, \infty)$

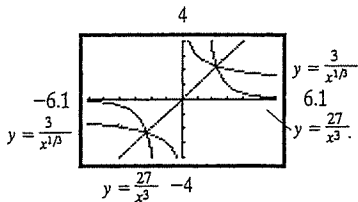


$$9. f^{-1}(x) = \frac{27}{x^3}$$

$$f(f^{-1}(x)) = \frac{3}{\left(\frac{27}{x^3}\right)^{1/3}} = x$$

$$f^{-1}(f(x)) = \frac{27}{\left(\frac{3}{x^{1/3}}\right)^3} = x$$

Domain  $f$  = Range  $f^{-1}$  = all real numbers except 0  
 Range  $f$  = Domain  $f^{-1}$  = all real numbers except 0



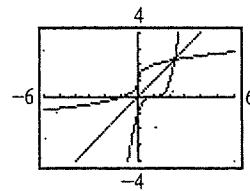
$$10. f^{-1}(x) = (x-1)^3$$

$$f(f^{-1}(x)) = ((x-1)^3 + 1)^{1/3} + 1 = x$$

$$f^{-1}(f(x)) = (x^{1/3} + 1 - 1)^3 = x$$

Domain  $f$  = Range  $f^{-1}$  =  $(-\infty, \infty)$

Range  $f$  = Domain  $f^{-1}$  =  $(-\infty, \infty)$



11. (a) 81 (b) 2 (c)  $\frac{1}{9}$  (d) -3

12. (a) 3 (b) 4 (c)  $\frac{1}{81}$  (d) -5

13.  $\log_5 z = 2$  14.  $\log_a m = 5$  15.  $5^{13} = u$  16.  $a^3 = 4$  17.  $\left\{x \mid x > \frac{2}{3}\right\}; \left(\frac{2}{3}, \infty\right)$  18.  $\left\{x \mid x > -\frac{1}{2}\right\}; \left(-\frac{1}{2}, \infty\right)$

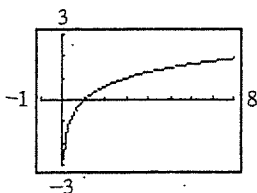
19.  $\left\{x \mid x < 1 \text{ or } x > 2\right\}; (-\infty, 1) \text{ or } (2, \infty)$  20.  $\{x \mid x < -3 \text{ or } x > 3\}; (-\infty, -3) \text{ or } (3, \infty)$  21. -3 22. 4 23.  $\sqrt{2}$  24. 0.1 25. 0.4

26.  $\sqrt{3}$  27.  $\log_3 u + 2 \log_3 v - \log_3 w$  28.  $8 \log_2 a + 2 \log_2 b$  29.  $2 \log x + \frac{1}{2} \log(x^3 + 1)$  30.  $2 \log_5(x+1) - 2 \log_5 x$

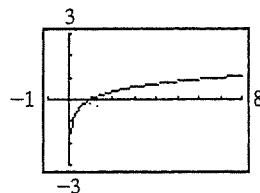
31.  $\ln x + \frac{1}{3} \ln(x^2 + 1) - \ln(x-3)$  32.  $2 \ln(2x+3) - 2 \ln(x-1) - 2 \ln(x-2)$  33.  $\frac{25}{4} \log_4 x$  34.  $\frac{13}{6} \log_3 x$  35.  $-2 \ln(x+1)$

36.  $\log\left(\frac{x-3}{x+4}\right)$  37.  $\log\left(\frac{4x^3}{[(x+3)(x-2)]^{1/2}}\right)$  38.  $\ln\left[\frac{16\sqrt{x^2+1}}{\sqrt{x(x-4)}}\right]$  39. 2.124 40. 4.392

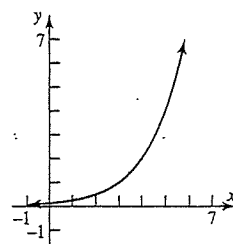
41.



42.



43.

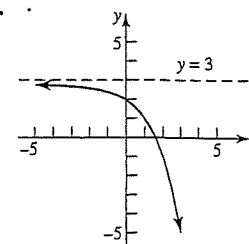


Domain:  $(-\infty, \infty)$

Range:  $(0, \infty)$

Horizontal asymptote:  $y = 0$

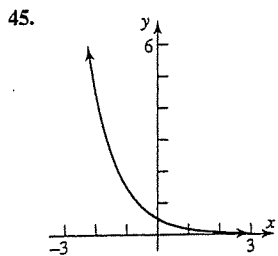
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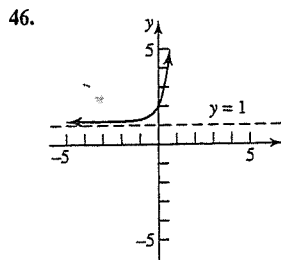
Domain:  $(-\infty, \infty)$

Range:  $(-\infty, 3)$

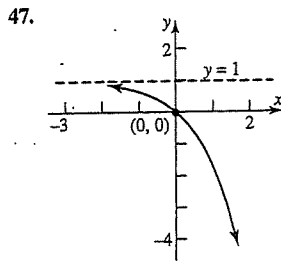
Horizontal asymptote:  $y = 3$



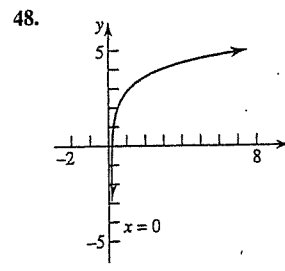
Domain:  $(-\infty, \infty)$   
 Range:  $(0, \infty)$   
 Horizontal asymptote:  $y = 0$



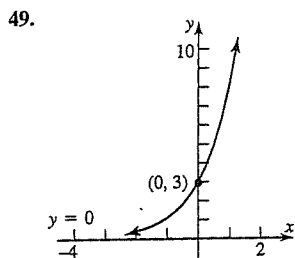
Domain:  $(-\infty, \infty)$   
 Range:  $(1, \infty)$   
 Horizontal asymptote:  $y = 1$



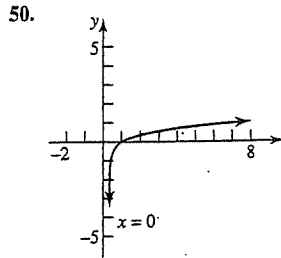
Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, 1)$   
 Horizontal asymptote:  $y = 1$



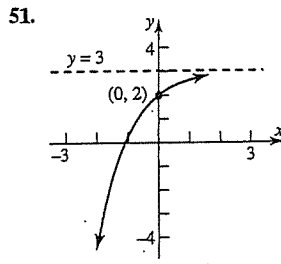
Domain:  $(0, \infty)$   
 Range:  $(-\infty, \infty)$   
 Vertical asymptote:  $x = 0$



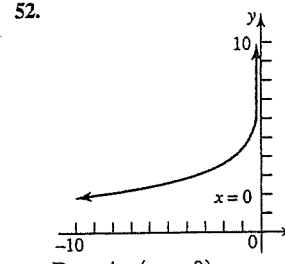
Domain:  $(-\infty, \infty)$   
 Range:  $(0, \infty)$   
 Horizontal asymptote:  $y = 0$



Domain:  $(0, \infty)$   
 Range:  $(-\infty, \infty)$   
 Vertical asymptote:  $x = 0$



Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, 3)$   
 Horizontal asymptote:  $y = 3$



Domain:  $(-\infty, 0)$   
 Range:  $(-\infty, \infty)$   
 Vertical asymptote:  $x = 0$

53.  $\left\{\frac{1}{4}\right\}$  54.  $\left\{-\frac{16}{9}\right\}$  55.  $\left\{\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right\}$  56.  $\left\{\frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right\}$  57.  $\left\{\frac{1}{4}\right\}$  58.  $\left\{\frac{1}{8}\right\}$  59.  $\left\{\frac{2\ln 3}{\ln 5 - \ln 3} \approx 4.301\right\}$   
 60.  $\left\{\frac{2(\ln 7 + \ln 5)}{\ln 7 - \ln 5} \approx 21.133\right\}$  61.  $\left\{\frac{12}{5}\right\}$  62.  $\{-2, 6\}$  63.  $\{83\}$  64.  $\left\{-\frac{1}{2}\right\}$  65.  $\left\{\frac{1}{2}, -3\right\}$  66.  $\{1\}$  67.  $\{-1\}$  68.  $\{3, 4\}$   
 69.  $\{1 - \ln 5 \approx -0.609\}$  70.  $\left\{\frac{1 - \ln 4}{2} \approx -0.193\right\}$  71.  $\left\{\frac{\ln 3}{3 \ln 2 - 2 \ln 3} \approx -9.327\right\}$  72.  $\left\{0, \frac{\ln 3}{\ln 2} \approx 1.585\right\}$