

Geometry (H)
Section 5.5 – Trapezoids

Name: KEY

Use the notes from section 5.5 to help you solve the following problems.

1. Draw the trapezoid described. If such a trapezoid cannot be drawn, explain why not.

a. with two right angles



b. with both bases shorter than the legs



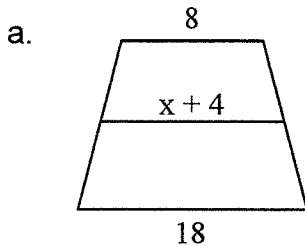
c. with congruent bases

Not possible; if bases are \cong , then legs will be \cong ; then not trap.

d. with three acute angles

Not possible; can draw a quad w/ 3 acute \angle s but both pairs opp sides not parallel.

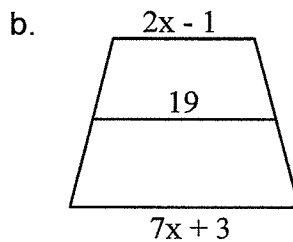
2. Each diagram shows a trapezoid and its median. Find the value of x .



$$x+4 = \frac{1}{2}(8+18)$$

$$x+4 = 13$$

$$x = 9$$

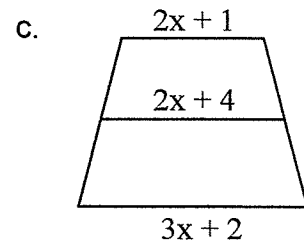


$$2(19) = 2x-1 + 7x+3$$

$$38 = 9x+2$$

$$36 = 9x$$

$$x = 4$$

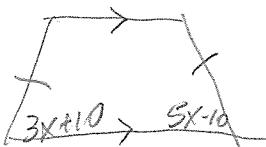


$$2x+4 = \frac{1}{2}(2x+1 + 3x+2)$$

$$4x+8 = 5x+3$$

$$5 = x$$

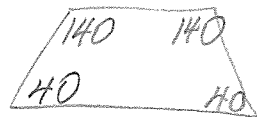
3. Two congruent angles of an isosceles trapezoid have measures $3x + 10$ and $5x - 10$. Find the value of x and then give the measures of all angles of the trapezoid.



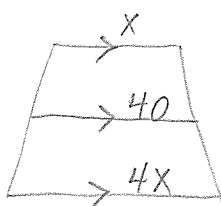
$$3x+10 = 5x-10$$

$$20 = 2x$$

$$10 = x$$



4. The length of one base of a trapezoid is four times the length of the other base. If the length of the median is 40, find the length of each base.



$$40 = \frac{1}{2}(x+4x)$$

$$80 = 5x$$

$$16 = x$$

$$\text{base 1} = 16$$

$$\text{base 2} = 64$$

5. In isosceles trap QRST with $\overline{QR} \parallel \overline{TS}$, $m\angle T = 3x + 12$, $m\angle S = 4x - 6$. Find $m\angle Q$.

$$3x + 12 = 4x - 6$$

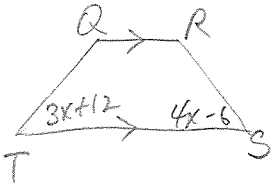
$$18 = x$$

$$3x + 12 =$$

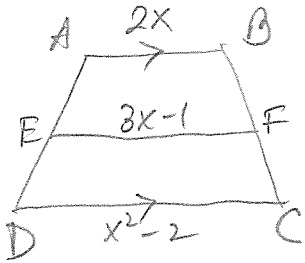
$$3(18) + 12 = 66$$

$$180 - 66 = 114$$

$$m\angle Q = 114$$



6. In trap ABCD with $\overline{AB} \parallel \overline{CD}$, and median \overline{EF} , $AB = 2x$, $EF = 3x - 1$, $DC = x^2 - 2$. Find the length of the median.



$$3x - 1 = \frac{1}{2}(x^2 - 2 + 2x)$$

$$6x - 2 = x^2 + 2x - 2$$

$$0 = x^2 - 4x$$

$$x(x - 4)$$

$$x = 0 \text{ (OMIT)} \quad | \quad x - 4 = 0$$

$$x = 4$$

$$AB = 8$$

$$EF = 11$$

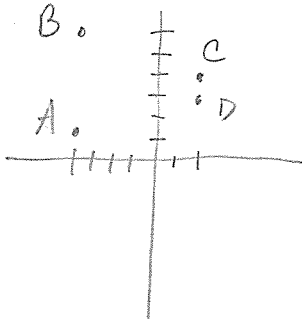
$$DC = 14$$

$$ck$$

$$11 = \frac{1}{2}(8 + 14)$$

7. Determine if the quadrilateral is a trapezoid. If it is, determine if it is an isosceles trapezoid.

A(-4,1), B(-4,6), C(2,4), D(2,3)



$$m_{AB} = \frac{6-1}{-4-4} = \frac{5}{-8} = \text{undefined}$$

$$m_{CD} = \frac{4-3}{2-2} = \frac{1}{0} = \text{undefined}$$

$AB \parallel CD$

$$m_{BC} = \frac{6-4}{-4-2} = \frac{2}{-6} = -\frac{1}{3}$$

$$m_{AD} = \frac{1-3}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$$

$AD \nparallel BC$

$$d_{BC} = \sqrt{(-4-2)^2 + (6-4)^2}$$

$$= \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$$

$$d_{AD} = \sqrt{(-4-2)^2 + (1-3)^2}$$

$$= \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$$

$AD \cong BC$

ABCD is an isosceles trapezoid.

8. Use the diagram at the right to solve the following problems.

a. If $AD = x + 3$, $BE = x + y$, and $CF = 36$, find x and y .

$$AD = \frac{1}{2} BE$$

$$x + 3 = \frac{1}{2}(x + y)$$

$$2x + 6 = x + y$$

$$x - y = -6$$

$$BE = \frac{1}{2}(AD + CF)$$

$$x + y = \frac{1}{2}(x + 3 + 36)$$

$$2x + 2y = x + 39$$

$$x + 2y = 39$$

$$x - y = -6$$

$$2x - 2y = -12$$

$$x + 2y = 39$$

$$3x = 27$$

$$x = 9$$

$$x + 3 = \frac{1}{2}(x + y)$$

$$9 + 3 =$$

$$24 = \frac{1}{2}(9 + y)$$

$$15 = y$$

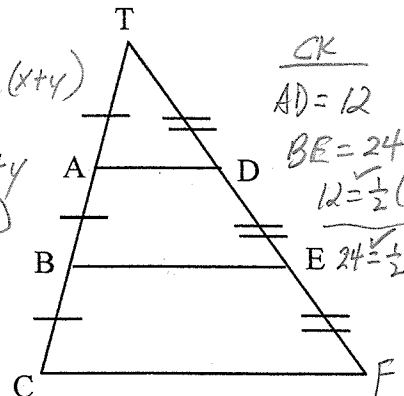
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$$AD = 12$$

$$BE = 24$$

$$12 = \frac{1}{2}(24)$$

$$E \quad 24 = \frac{1}{2}(42)$$



b. If $AD = x + y$, $BE = 20$ and $CF = 4x - y$, then find CF .

$$AD = \frac{1}{2} BE$$

$$x + y = \frac{1}{2}(20)$$

$$x + y = 10$$

$$y = 2$$

$$BE = \frac{1}{2}(AD + CF)$$

$$20 = \frac{1}{2}(x + y + 4x - y)$$

$$40 = 5x$$

$$8 = x$$

$$CF = 4x - y$$

$$CF = 30$$

$$AD = 10$$

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$$10 = \frac{1}{2}(20)$$

$$10 = 10$$

$$20 = \frac{1}{2}(10 + 30)$$

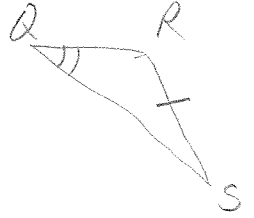
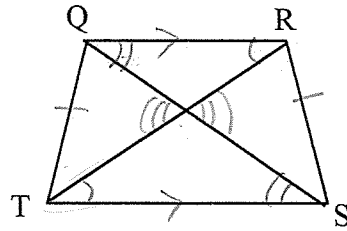
$$20 = 20$$

9. Write a flow proof for the following theorem.

The diagonals of an isosceles trapezoid are congruent.

Given: Isos trap TQRS with $\overline{QR} \parallel \overline{TS}$

Prove: $\overline{TR} \cong \overline{QS}$ (Prove $\triangle TQR \cong \triangle SRQ$
or $\triangle QTS \cong \triangle RST$)



- ① Isos trap TQRS w/ $\overline{QR} \parallel \overline{TS}$ → ② $\overline{QT} \cong \overline{RS}$
 → ③ $\angle TQR \cong \angle SRQ$
 ④ $\overline{QR} \cong \overline{QR}$ } → ⑤ $\triangle TQR \cong \triangle SRQ$

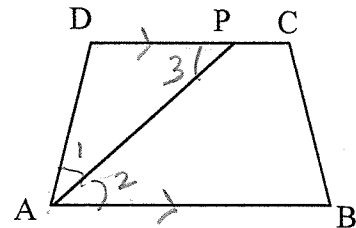
⑥ $\overline{TR} \cong \overline{QS}$

- ① Given
 ② In isos trap, legs are \cong .
 ③ In isos trap, base \angle s \cong .
 ④ Reflexive property
 ⑤ SAS \cong SAS
 ⑥ CPCTC

10. Write a flow proof for each the following.

a. Given: Trap ABCD with bases \overline{AB} and \overline{CD}
 \overline{AP} bisects $\angle A$

Prove: $\triangle APD$ is isosceles



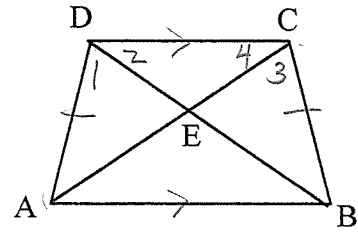
- ① Trap ABCD w/ bases \overline{AB} & \overline{CD} → ② $\overline{AB} \parallel \overline{CD}$ → ③ $\angle 3 \cong \angle 2$
 ④ \overline{AP} bisects $\angle A$ → ⑤ $\angle 1 \cong \angle 2$ } → ⑥ $\angle 1 \cong \angle 3$

→ ⑦ $\overline{AD} \cong \overline{DP}$ → ⑧ $\triangle APD$ is isosceles.

- ① Given
 ② In a trap, bases \parallel .
 ③ 2 \parallel lines → alt. int. \angle s \cong
 ④ Given
 ⑤ Def of \angle bisector
 ⑥ Transitive Prop.
 ⑦ In a \triangle , if 2 \angle s \cong , then sides opp \cong .
 ⑧ Def of isos. \triangle .

b. Given: Isos. Trap ABCD with bases \overline{AB} and \overline{CD}

Prove: $\triangle CDE$ is isosceles



① Isos. trap. ABCD w/ bases \overline{AB} & \overline{CD} . \rightarrow ② $\overline{AD} \cong \overline{BC}$
 \rightarrow ③ $\overline{BD} \cong \overline{AC}$
 ④ $\overline{AB} \cong \overline{AB}$ } \rightarrow ⑤ $\triangle ADB \cong \triangle BCA \rightarrow$ ⑥ $\angle 1 \cong \angle 3$ \rightarrow ⑩ $\angle 2 \cong \angle 4$

⑦ $\angle ADC \cong \angle BCD$ } \rightarrow ⑧ $\angle ADC \cong \angle 1 + \angle 2$
 $\angle BCD \cong \angle 3 + \angle 4$ } \rightarrow ⑨ $\angle 1 + \angle 2 \cong \angle 3 + \angle 4$ \rightarrow ⑪ $\overline{DE} \cong \overline{CE}$

⑩ $\triangle CDE$ is isosceles.

⑫ Def of isos. \triangle .

① Given
 ② In an isos. trap, legs \cong .
 ③ In an isos. trap, diagonals \cong .
 ④ Reflexive Prop.
 ⑤ SSS \cong SSS
 ⑥ CPCTC
 ⑦ In isos trap, base \angle s \cong .
 ⑧ Angle Addition Prop.
 ⑨ Substitution
 ⑩ Subtraction Postulate
 ⑪ If 2 \angle s \cong , sides opp \cong

11. Determine if the following statements are sometimes, always or never true. Justify your answer.

a. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles.

Always True It can be proven that $\triangle ABC \cong \triangle BAD$ by SAS. Therefore, legs \cong by CPCTC.

b. If a quadrilateral has more than two right angles, then it is not a trapezoid.

Always true. If a quad has 3 right \angle s, the 4th \angle will be 90. This would make it a rectangle.

c. The opposite angles of an isosceles trapezoid are supplementary.

Always true. If base $\angle = x$, then the 2 base \angle s = $2x$. The top 2 \angle s = $360 - 2x$. If $(360 - 2x) \div 2$, each opp. $\angle = 180 - x$.

d. The diagonals of an isosceles trapezoid bisect each other.

Never

e. If the consecutive angles of a quadrilateral are supplementary, it is a trapezoid.

Never. Will be a \square . $360 - [2(180 - x) + x] = ?$
 $360 - (360 - x) = ?$
 $360 - 360 + x = ?$ $? = x$