

answer key with it.

ADVANCED ALGEBRA II (H)  
CHAPTER 10 REVIEW

NAME \_\_\_\_\_  
DATE \_\_\_\_\_ PD \_\_\_\_\_

1. How many roots does  $9x - 5x^2 + x^3 - 45$  have? What are they?

$$(x^3 - 5x^2) + (9x - 45)$$

$$x^2(x-5) + 9(x-5)$$

$$x^2 = -9$$

$$x = \pm 3i$$

$$\boxed{\text{roots: } 3i, -3i, 5}$$

$$(x^2 + 9)(x - 5) \quad \leftarrow \text{factored form}$$

2. Find a polynomial, in lowest degree, which has  $1 - \sqrt{2}$ ,  $i$ , and  $1 + 4i$  as roots. (no conjugates)

not lowest degree  $\rightarrow$  multiply out  
point to examples  
lowest degree

$$(x - (1 - \sqrt{2}))(x - i)(x - (1 + 4i))$$

$$\boxed{(x - 1 + \sqrt{2})(x - i)(x - 1 - 4i) = P(x)}$$

$\leftarrow$  factored form.

3. Find a polynomial with rational coefficients, in lowest degree, with  $1 + i$  and  $2 - \sqrt{5}$  as roots.

Do use conjugates  $\leftarrow$

$$P(x) = (x - (1 + i))(x - (1 - i))(x - (2 - \sqrt{5}))(x - (2 + \sqrt{5})) \rightarrow P(x) = 2x^2 - 6x + 1$$

$$\boxed{P(x) = (x - 1 - i)(x - 1 + i)(x - 2 + \sqrt{5})(x - 2 - \sqrt{5})}$$

4. Find all roots of:  $100 + 4x^2 + 100x + 4x^3 + 25x^2 + x^4 = 0$

$$x^4 + 4x^3 + 29x^2 + 100x + 100 = 0$$

$$b: \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm 25, \pm 50, \pm 100$$

$$P(-2) = 0 \quad \underline{-2} \quad \begin{array}{r|rrrrrr} 1 & 1 & 4 & 29 & 100 & 100 \\ & & -2 & -4 & -50 & -100 \\ \hline & 1 & 2 & 25 & 50 & 0 \\ & & -2 & 0 & -50 & 0 \\ \hline & 1 & 0 & 25 & 0 & 0 \\ & & x^2 + 25 & & & \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 25 & 50 & 0 \\ & & -2 & 0 & -50 & 0 \\ \hline & 1 & 0 & 25 & 0 & 0 \\ & & x^2 + 25 & & & \end{array}$$

$$x^2 = -25$$

$$x^2 = (5i)^2$$

$$(x + 5i)(x - 5i)$$

$$\boxed{\text{roots: } -2, -2, -5i, 5i}$$

5. Find a polynomial, in lowest degree, which has  $4 - \sqrt{3}$  and  $9 - 2i$  as roots. no conjugates

$$(x - (4 - \sqrt{3}))(x - (9 - 2i))$$

$$\boxed{(x - 4 + \sqrt{3})(x - 9 + 2i) = P(x)}$$

6. Find the factors of a polynomial with rational coefficients, in lowest degree, with  $5 - 12i$  and  $3 + \sqrt{2}$  as roots.

$$P(x) = (x - (5 - 12i))(x - (5 + 12i))(x - (3 + \sqrt{2}))(x - (3 - \sqrt{2}))$$

$$\boxed{P(x) = (x - 5 + 12i)(x - 5 - 12i)(x - 3 - \sqrt{2})(x - 3 + \sqrt{2})}$$

7. Factor completely:  $x^3 - 5x^2 + 9x - 5 = 0$

$b = \pm 1, \pm 5$

$b = 1$

$x = 1$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 9 & -5 \\ & & 1 & -4 & 5 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$x^2 - 4x + 5$

$$x = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{4 \pm \sqrt{-4}}{2}$$

$$= 2 \pm i$$

$(x-1)(x-2+i)(x-2-i)$  factors

8. Find all the roots of:  $x^4 + x^3 + x^2 - 9x - 10 = 0$

$\pm 1, \pm 2, \pm 5, \pm 10$

$b = 2 \rightarrow$

$$\begin{array}{r|rrrrrr} 2 & 1 & 1 & 1 & -9 & -10 \\ & & 2 & 6 & 14 & 10 \\ \hline & 1 & 3 & 7 & 5 & 0 \end{array}$$

$x^3 + 3x^2 + 7x + 5$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 7 & 5 \\ & & -1 & -2 & -5 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

$x^2 + 2x + 5$

$$x = \frac{-2 \pm \sqrt{4 - 4(5)}}{2}$$

$$= -1 \pm 2i$$

roots  $\{-1, 2, -1+2i, -1-2i\}$

9. Find a polynomial, in lowest degree, which has  $3 - \sqrt{5}$  and  $8 + 3i$  as roots.

$P(x) = (x - 3 + \sqrt{5})(x - 8 - 3i)$

10. Find factors of a polynomial with rational coefficients, in lowest degree, with  $3 - 5i$  and  $8 + \sqrt{5}$  as roots.

$P(x) = (x - 3 - 5i)(x - 3 + 5i)(x - 8 - \sqrt{5})(x - 8 + \sqrt{5})$

11. Use  $P(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$  to answer the following:

The roots of  $P(x)$  are  $i, -i, 3, 2$

a) Which of these roots, if any, are rational?  $3$  and  $2$

b) Which of these roots, if any, are not rational?  $i, -i$

12. Use  $P(x) = x^5 - 2x^4 - 3x^3 + 6x^2 - 4x + 8$  to answer the following:

a) How many roots does this polynomial have? How do you know?

b) Given the roots:  $-2, -i$ , and  $2$  (multiplicity 2), how many more roots does  $P(x)$  have? Find them.

one more; it must be  $-i$ .

c) How did you find the missing root(s) in part b)?

The coefficients are all rational. So, if one root is complex, then there must be another root that is complex, and is its conjugate.

$$x = \frac{6 \pm \sqrt{36-52}}{2}$$

$$= \frac{6 \pm \sqrt{-16}}{2}$$

$$= 3 \pm 2i$$

13. Factor completely:  $z^3 - 5z^2 + 7z + 13 = 0$

$$b = \pm 1, \pm 13$$

$$P(-1) = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 7 & 13 \\ & & -1 & 6 & -13 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$x^2 - 6x + 13$$

$$\text{Factors: } (z+1)(z-3-2i)(z-3+2i)$$

14. Find all the roots of  $x^4 - 3x^3 + 7x^2 + 21x - 26 = 0$ .

$$b = \pm 1, \pm 2, \pm 13, \pm 26$$

$$P(1) = 0$$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 7 & 21 & -26 \\ & & 1 & -2 & 5 & 26 \\ \hline & 1 & -2 & 5 & 26 & 0 \end{array}$$

$$x^3 - 2x^2 + 5x + 26$$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 5 & 26 \\ & & -2 & 8 & -26 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

$$x^2 - 4x + 13$$

$$x = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= 2 \pm 3i$$

$$\text{roots } \left\{ 1, -2, 2+3i, 2-3i \right\}$$

15. If  $P(x) = x^3 - x^2 + x - 1$  and one of the roots is equal to 1, find the other roots of  $P(x)$ .

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 1 & -1 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$x^2 + 1$$

$$x^2 - -1 \rightarrow x^2 - \sqrt{-1}^2$$

$$(x+i)(x-i)$$

$$\text{roots: } 1, i, -i$$

$$\text{other roots: } 1, i, -i$$

16. Find a polynomial of degree 5 with -1 as a root (multiplicity 3), 0 as a root (multiplicity 1), and 4 as a root (multiplicity 1). \*does not say "lowest degree"

$$P(x) = (x+1)^3 x(x-4)$$

$$= (x^3 + 3x^2 + 3x + 1)(x^2 - 4x)$$

$$= x^5 + 3x^4 + 3x^3 + x^2 - 4x^4 - 12x^3 - 12x^2 - 4x$$

$$P(x) = x^5 - x^4 - 9x^3 - 11x^2 - 4x$$

17. If  $P(x) = x^4 - 5x^3 + 10x^2 - 20x + 24$ , then find all the roots given that  $2i$  is a root.

real coeff  $\Rightarrow$  complex conjugate

$$\begin{array}{r|rrrrr} 2i & 1 & -5 & 10 & -20 & 24 \\ & & 2i & -4-10i & 20+12i & -24 \\ \hline & 1 & -5+2i & 6-10i & 12i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2i & 1 & -5+2i & 6-10i & 12i \\ & & -2i & 10i & -12i \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$\text{roots: } 2i, -2i, 3, 2$$

18. Find all the roots of  $x^3 - 7x^2 + 17x - 15$  if  $2-i$  is one root.

Done in 2 ways:

$$P(x) = (x-2-i)(x-2+i) \cdot Q(x)$$

$$= ((x-2)^2 - i^2) \cdot Q(x)$$

$$= (x^2 - 4x + 4 + 1) \cdot Q(x)$$

$$= (x^2 - 4x + 5) \cdot Q(x)$$

$$\begin{array}{r} \text{quotient } x^2 - 4x + 5 \\ \hline x^3 - 7x^2 + 17x - 15 \\ + x^3 + 4x^2 + 5x \\ \hline -3x^2 + 12x - 15 \\ -3x^2 + 12x - 15 \\ \hline 0 \end{array}$$

$$\text{OR } \begin{array}{r|rrrr} 2-i & 1 & -7 & 17 & -15 \\ & & 2-i & 3i-11 & \\ \hline & 1 & -5-i & 3i+6 & \end{array}$$

$$\text{roots } (2-i)(2+i)(3)$$

19. Find all the possible rational roots of:  $P(x) = x^4 + 2x^3 + 5x^2 + 34x + 30$

$$b = \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$$

actual:  $-1, -3, 1+3i, 1-3i$

20. Find a polynomial of degree 3 with roots 0, 1 and  $i$ .

$$P(x) = (x-0)(x-1)(x-i)$$

$$x(x^2 - x - ix + i)$$

$$x^3 - x^2 - ix^2 + ix \rightarrow P(x) = x^3 - (1+i)x^2 + ix$$

21.  $(2x^4 + 7x^3 + x - 12) \div (x+3)$

$$\begin{array}{r|rrrrr} -3 & 2 & 7 & 0 & 1 & -12 \\ & & -6 & -3 & 9 & -30 \\ \hline & 2 & 1 & -3 & 10 & -42 \end{array}$$

$$P(x) = 2x^3 + x^2 - 3x + 10 - \frac{42}{x+3}$$

22.  $P(x) = x^3 + 7x^2 - 12x - 3$ . Find  $P(-3)$ ,  $P(-2)$ , and  $P(1)$ .

$$\begin{array}{r|rrrr} -3 & 1 & 7 & -12 & -3 \\ & & -3 & -12 & 72 \\ \hline & 1 & 4 & -24 & 69 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 7 & -12 & -3 \\ & & -2 & -10 & 44 \\ \hline & 1 & 5 & -22 & 41 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & 7 & -12 & -3 \\ & & 1 & 8 & -4 \\ \hline & 1 & 8 & -4 & -7 \end{array}$$

$$P(-3) = 69$$

$$P(-2) = 41$$

$$P(1) = -7$$

For questions 23-25, find the roots stating the multiplicity of each.

23.  $(x^2 - 7x + 12)^2 = 0$

$$[(x-4)(x-3)]^2 = 0$$

$$(x-4)^2(x-3)^2 = 0$$

$$3 \text{ (multiplicity 2)}, 4 \text{ (multiplicity 2)}$$

24.  $x^3(x-1)(x+4) = 0$

$$0 \text{ (multiplicity 3)}, 1 \text{ (multiplicity 1)}, -4 \text{ (multiplicity 1)}$$

25.  $-8(x-3)^2(x+4)^3x^4 = 0$

$$0 \text{ (multiplicity 4)}, -4 \text{ (multiplicity 3)}, 3 \text{ (multiplicity 2)}$$

Just a division problem