



- R6. a.  $P(x) = 3x^3 - 10x^2 + 2x + 5$   
 b.  $-63$   
 c.  $1$

- R6. A cubic function  $P(x)$  contains the ordered pairs  $(-1, -10)$ ,  $(2, -7)$ ,  $(3, 2)$  and  $(4, 45)$ .  
 a. Find the particular equation.  
 b. Find  $P(-2)$ .  
 c. Find one integer zero of  $P(x)$ .

Concepts Problems

- C1. a–c. See Additional Answers.  
 C2. See Additional Answers.  
 C3. See Additional Answers.  
 C4. a.  $(0, 0)$ ,  $(16, 1)$ ,  $(20, 2)$ ,  $(20, 5)$

CONCEPTS PROBLEMS

- C1. Sketch the graph of  
 a. a seventh degree function,  
 b. a quartic function with two distinct real zeros and two non-real complex zeros,  
 c. a polynomial function with two real zeros at the same point.  
 C2. Prove that

$$\frac{a - bi}{b + ai} = -i.$$

- ~~C3.~~ You have learned that if  $s_1$  and  $s_2$  are solutions of a quadratic equation  $ax^2 + bx + c = 0$ , then  $s_1 + s_2 = -\frac{b}{a}$ . If  $s_1$ ,  $s_2$ , and  $s_3$  are solutions of a cubic equation  $ax^3 + bx^2 + cx + d = 0$ , prove that

$$s_1 + s_2 + s_3 = -\frac{b}{a}.$$

Then find the sum of the solutions of the equation

$$5x^3 + 11x^2 - 13x + 47 = 0.$$

- C4. **Catastrophe Theory Problem** According to “catastrophe theory” (see for example *Scientific American*, April, 1976), when a person is under certain kinds of stress, the amount of food eaten is *not* a simple function of how hungry he or she is (see Figure 10-7). As hunger increases, food consumption increases slightly, but not enough to satisfy the hunger. Then at a certain point a “catastrophe” happens, and the person starts “gorging” himself or herself. The hunger decreases, but remains high until a “reverse catastrophe” happens, and the person starts “fasting” again. Suppose that Juanita Lott is under this kind of stress. When her hunger is 16 units, she eats only 1000 Calories per day. At the catastrophe point, 20 hunger units, she jumps from 2000 to 5000 Calories per day. The point  $(0, 0)$  is also on the graph.

Let  $x$  be the number of units of hunger.

Let  $y$  be the number of *thousands* of Calories per day.

- a. Write the given information as *four* ordered pairs,  $(x, y)$ .

$$y = ax^3 + bx^2 + cx + d$$

$$\begin{aligned}
(4) & \rightarrow -10 = -1a + b - c + d \\
(3) & \rightarrow (-7 = 8a + 4b + 2c + d) \text{ (1)} \\
(3) & \rightarrow (2 = 27a + 9b + 3c + d) \text{ (2)} \\
& \rightarrow 45 = 64a + 16b + 4c + d \text{ (2)}
\end{aligned}$$

$$\begin{aligned}
(1) & \rightarrow \begin{matrix} -10 = -a + b - c + d \\ + 7 = 8a + 4b + 2c + d \end{matrix} \rightarrow \begin{matrix} -3 = -9a - 3b - 3c \\ -1 = -3a - b - c \end{matrix} \left. \vphantom{\begin{matrix} -10 = -a + b - c + d \\ + 7 = 8a + 4b + 2c + d \end{matrix}} \right\} 42 = 34a + 6b \\
(2) & \rightarrow \begin{matrix} 45 = 64a + 16b + 4c + d \\ - 2 = 27a + 9b + 3c + d \end{matrix} \rightarrow 43 = 37a + 7b + c
\end{aligned}$$

$$\begin{aligned}
42 &= 34a + 6b \\
6 &= -18a - 6b \\
\hline
48 &= 16a \\
3 &= a
\end{aligned}$$

$$\begin{aligned}
(3) & \rightarrow \begin{matrix} 2 = 27a + 9b + 3c + d \\ + 7 = 8a + 4b + 2c + d \end{matrix} \rightarrow \begin{matrix} 9 = 19a + 5b + c \\ -11 = -13a - 3b - c \end{matrix} \\
& \rightarrow 9 = 19a + 5b + c \\
& \rightarrow (-2 = 6a + 2b) \cdot (-3)
\end{aligned}$$

$$\begin{aligned}
(4) & \rightarrow \begin{matrix} 45 = 64a + 16b + 4c + d \\ + 10 = 1a + b + c + d \end{matrix} \\
& \rightarrow 55 = 65a + 15b + 5c \\
& \rightarrow 11 = 13a + 3b + c
\end{aligned}$$