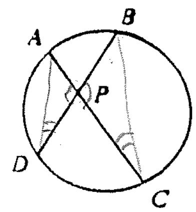
NOTES

Geometry (Honors)

Section 9.7 – Segments of Chords, Secants & Tangents

Thm 9.11 – If two chords intersect in a circle, then the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the second chord.

Given: $\overline{AC}$ and $\overline{BD}$ are chords intersecting at P



Prove: $AP \cdot PC = BP \cdot PD$

$\frac{AP}{BP} = \frac{PD}{PC}$

① Draw $\overline{AD}$ & $\overline{BC}$.

② $\angle ADB \cong \angle ACB$

③ $\angle APD \cong \angle BPC$

$\rightarrow$ ④ $\triangle APD \sim \triangle BPC \rightarrow$ ⑤ $\frac{AP}{BP} = \frac{PD}{PC} \rightarrow$ ⑥ $AP \cdot PC = BP \cdot PD$

① 2 pts. determine a line.

② Inscribed $\angle$s that intercept same arc are $\cong$.

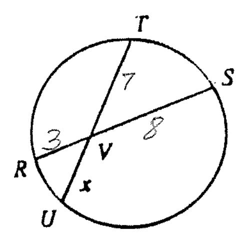
③ Vertical $\angle$s $\cong$.

④ A.A. ~ A.A

⑤ ~$\triangle$s, corresp. parts proportional.

⑥ means - extremes property.

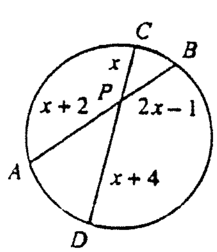
Example 1: Find x if RV = 3, TV = 7, and SV = 8



$3(8) = 7x$

$\boxed{\frac{24}{7} = x}$

Example 2: Find the lengths of chords $\overline{AB}$ and $\overline{CD}$.



$(x+2)(2x-1) = x(x+4)$

$2x^2 + 3x - 2 = x^2 + 4x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$, $x = -1$ OMIT b/c $CP \neq -1$.

$\boxed{\begin{array}{l} AB = 4 + 3 = 7 \\ CD = 2 + 6 = 8 \end{array}}$

CK

CD AB

$2(6) = 4(3)$

$12 = 12$

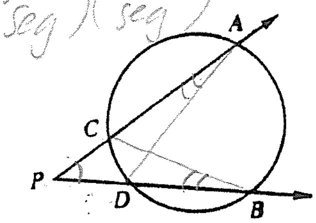
Point out :

Thm 9.12 – If two secant segments are drawn to a circle from an exterior point, then the product of the lengths of one secant segment and its external segment equal the product of the lengths of the other secant segment and its external segment.

$\binom{sec}{seg}\binom{ext.}{seg} = \binom{sec}{seg}\binom{ext}{seg}$

Given: $\overline{PA}$ and $\overline{PB}$ are secants

Prove: $PD \cdot PB = PC \cdot PA$



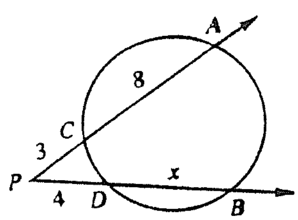
$\rightarrow \dfrac{PD}{PC} = \dfrac{PA}{PB} \rightarrow$ Show $\triangle PDA \sim \triangle PCB$

① Draw $\overline{AD}$ & $\overline{BC}$.

② $\angle PAD \cong \angle PBC$
③ $\angle P \cong \angle P$ $\Big\} \rightarrow$ ④ $\triangle PDA \sim \triangle PCB \rightarrow$ ⑤ $\dfrac{PD}{PC} = \dfrac{PA}{PB} \rightarrow$ ⑥ $PD \cdot PB = PC \cdot PA$

① 2 pts determine a line.
② 2 inscribed ∡s that intercept same arc are ≅.
③ Reflexive Prop.
④ AA ~ AA
⑤ ~△s → corresp. parts proportional.
⑥ Means – extremes property.

Example 3: Find x in the figure below.



$11(3) = (4+x)4$

$33 = 16 + 4x$

$17 = 4x$

$\dfrac{17}{4} = x$

CK

$33 = 8\frac{1}{4}(4)$

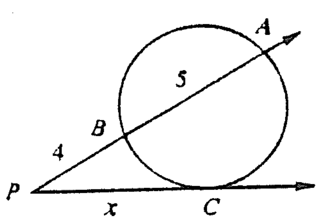
$33 = 32 + 1$

$33 = 33$ ✓

Thm 9.13 – If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the length of the tangent segment equals the product of the lengths of the secant segment and is external secant segment.

$tan^2 = \binom{sec.}{seg}\binom{ext.}{sec.}$

Example 4: Find x in the figure below.



$x^2 = 4(9)$

$x^2 = 36$

$x = 6$