KEY

Geometry (H)

Section 12.1 & 12.3 – Area & Volume of Prisms and Cylinders

1. The length of a rectangular solid is twice the width and the height is three times the width. If the volume is 162 cm^3, find the total area of the solid.

Let w = width
2w = length
3w = ht.

$V = Bh$
$162 = 6w^3$
$27 = w^3$
$3 = w$
$6 = l$
$9 = ht.$

$TA = LA + 2B$
$18(9) + 2(18)$
$162 + 36$
$TA = 198$ sq. cm

2. A prism has square bases with edges that are three times as long as the height. The prism's total area is 750 m^2. Find the volume.

$e = ht$
$3e = edge$

$TA = LA + 2B$
$= 12e(e) + 2(9e^2)$
$750 = 12e^2 + 18e^2$
$750 = 30e^2$
$5 = e$

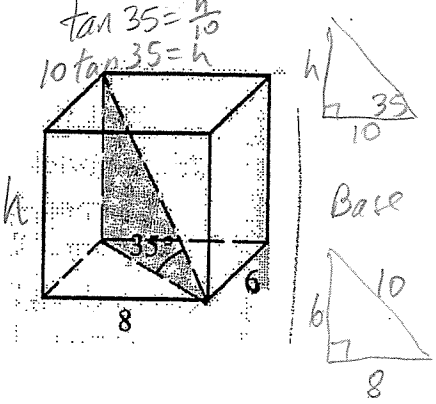
$V = 5^3$
$= 125$ cu. m.

3. A diagonal of a box forms a 35° angle with a diagonal of the base, as shown. Find the volume of the box. Round your answer to the nearest tenth.

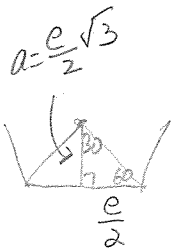
$V = Bh$ ← need h.
$B = 6 \times 8 = 48$

$\tan 35 = \frac{h}{10}$
$10 \tan 35 = h$

$V = 48(10 \tan 35)$
≈ 336.1 sq. units



4. The volume of a hexagonal prism is $180\sqrt{3}$. The height is 6. Find the lateral area.



$V = Bh$
$180\sqrt{3} = B6$
$30\sqrt{3} = B$

$B = \frac{1}{2}ap$
$p = 6e$
$30\sqrt{3} = \frac{1}{2}\left(\frac{e\sqrt{3}}{2}\right)(6e)$
$30 = \frac{3e^2}{2}$
$60 = 3e^2$
$20 = e^2$
$2\sqrt{5} = e$

$LA = ph$
$= 12\sqrt{5}(6)$
$LA = 72\sqrt{5}$

5. Find the total area and volume of a half cylinder with a radius of 6 and height of 20.

$\frac{1}{2}(TA) = \frac{1}{2}(LA + 2B) + A_{rectangle}$

$= \frac{1}{2}(2\pi(6)(20) + 2\pi 6^2) + 12(20)$

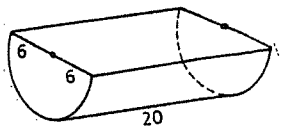
$= 120\pi + 36\pi + 240$

$TA = \boxed{156\pi + 240}$

$\frac{1}{2}V = (\pi r^2 h)\frac{1}{2}$

$= \pi 6^2 (20)\frac{1}{2}$

$V = \boxed{360\pi}$



Rect.

6. A wedge of cheese is cut from a cylindrical block. Find the volume and total area of the wedge.

$V = \pi r^2 h$

$V_{wedge} = \frac{40}{360}(\pi 20^2)(15)$

$= \frac{1}{9}\pi 6000$

$= \frac{6000\pi}{9}$

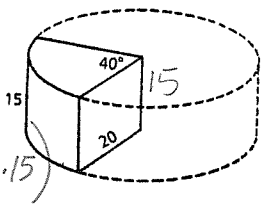
$\boxed{V = \frac{2000\pi}{3}}$

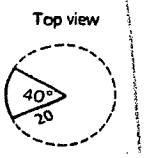
$TA = \frac{1}{9}(LA + 2B) + 2A_{rect.}$

$= \frac{1}{9}\left[(2\pi(20)(15) + 2(\pi 20^2)\right] + 2(20 \cdot 15)$

$= \frac{1}{9}[600\pi + 800\pi +] + 600$

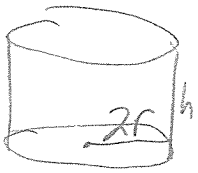
$\boxed{\frac{1400\pi}{9} + 600}$





7. If the radius of a cylinder is doubled, is its volume doubled? Show support for your answer.

$V = \pi r^2 h$



h stays same.

$V = \pi(2r)^2 h$

$= 4\pi r^2 h$

The volume is quadrupled.

8. A case is tightly packed with six cylindrical cans. Find the ratio of the volume of the box to the combined volumes of the cans.

$V_{box} = (4r)(6r)h$

$= 24r^2 h$

$V_{can} = \pi r^2 h$

$V_{6\ cans} = 6\pi r^2 h$

Let $4r$ = width of box

$6r$ = length of box

h = ht of box & cans

Ratio of $V_{box} : V_{cans}$

$24r^2 h : 6\pi r^2 h$

$24 : 6\pi$

$\boxed{4 : \pi}$