Geometry (H)
Section – Spheres

Name: KEY

1. Find the area and volume of a sphere with radius 5 cm.

$A = 4\pi r^2$
$= 4\pi 5^2 = \boxed{100\pi \text{ sq. cm}}$

$V = \frac{4}{3}\pi r^3$
$\frac{4}{3}\pi 5^3 = \boxed{\frac{500\pi}{3} \text{ cu. cm}}$

2. Find the radius of a sphere with an area of 64π.

$A = 4\pi r^2$ $\quad 64\pi = 4\pi r^2$
$16 = r^2$
$4 = r$
$\boxed{r = 4}$

3. Find the radius of a sphere with a volume of 288π.

$V = \frac{4}{3}\pi r^3$ $\quad 288\pi = \frac{4}{3}\pi r^3$
$216 = r^3 \longrightarrow \boxed{r = 6}$

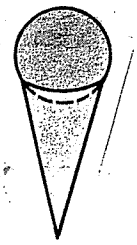
4. A scoop of ice cream with diameter 6 cm is placed in an ice cream cone with diameter 5 cm and height 12 cm. Is the cone big enough to hold all the ice cream if it melts?

$r = \frac{5}{2}$ $\quad r = 3$

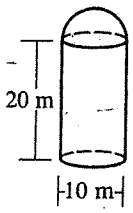
Cone
$V = \frac{1}{3}Bh$
$\frac{1}{3}\pi(2.5)^2(12)$
$\frac{75\pi}{3} = \boxed{25\pi}$

Ice Cream
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi 3^3$
$= 36\pi$

Since $25\pi < 36\pi$, no, the cone is not big enough.



5. A silo of a barn consists of a cylinder capped by a hemisphere, as shown. Find the volume of the silo.



$V_{silo} = V_{\frac{1}{2}\text{sphere}} + V_{\text{cylinder}}$
$= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \pi r^2 h$
$= \frac{1}{2}\cdot\frac{4}{3}\pi 5^3 + \pi 5^2(20)$
$= \frac{250\pi}{3} + 500\pi = \boxed{\frac{1750\pi}{3}}$

6. A solid metal ball with radius 8 cm is melted down and recast as a solid cone with the same radius. What is the height of the cone?

$V_{ball} = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi 8^3$
$= \frac{2048\pi}{3}$

$V_{cone} = \frac{1}{3}\pi r^2 h$
$\frac{2048\pi}{3} = \frac{1}{3}\pi 8^2 h$
$2048 = 64h$
$\text{ft. } 32 = h$
$\longrightarrow \boxed{h = 32}$

7. A spherical water tank is 40 in diameter. How many gallons of paint will be needed to cover the tank with one coat? One gallon of paint covers 300 sq. ft.

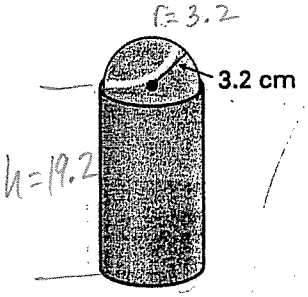
$TA = 4\pi r^2$
$= 4(3.14)(20)^2$
$= 5024$ sq. ft.

Inches

$\frac{5024 \text{ sq.}}{300} \approx \boxed{16.75 \text{ gallons}}$

1 sq ft
$= 12 \times 12$
$= 144$ inches

8. Tennis balls are often sold in cans of three. Assuming that the balls are packed tightly so that they touch the lateral side and the bases, determine the amount of volume in the can that is not taken up by the tennis balls.



$V_{can} = Bh$
$= \pi(3.2)^2(19.2)$
$= 196.608\pi$

$V_{ball} = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(3.2)^3$
$= 43.6907$
$\times \quad 3$
131.0721

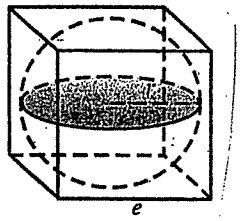
$V_{free} = 65.536\pi$

9. A sphere of radius r is inscribed in a cube of edge length e.

$l = w = h = e$

a. The longest diagonal of the cube is $6\sqrt{3}$. Find the volume of the sphere.

$d = \sqrt{l^2 + w^2 + h^2}$
$6\sqrt{3} = \sqrt{3e^2}$
$6\sqrt{3} = \sqrt{3}\sqrt{e^2}$
$e^2 = 36$
$e = 6$
$r = 3$
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi 3^3$
$V = 36\pi$ Sphere



b. What is the ratio of the volume of the sphere to the volume of the cube?

$V_{cube} = 6^3$
$= 216$

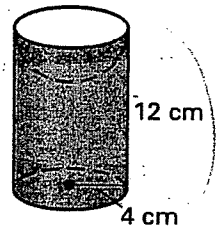
sphere : cube
$6^2\pi : 6^3$
$\pi : 6$

OR

$\frac{4}{3}\pi r^3 : (2r)^3$
$\frac{4}{3}\pi r^3 : 8r^3$
$\pi : 6$

10. Find the area and volume of each figure.

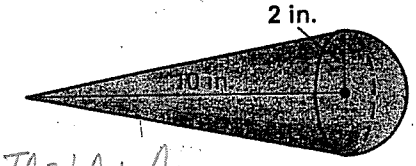
a.



$V = V_{cylinder} - V_{\frac{1}{2}sphere}$
$\pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$\pi 16(12) - \frac{2}{3}\pi 4^3$
$= 192\pi - \frac{128\pi}{3}$
$= \frac{448\pi}{3}$

$TA = A_{cylinder} + A_{\frac{1}{2}sphere}$
$2\pi rh + \pi r^2 + \frac{1}{2}(4\pi r^2)$
$2\pi(4)(12) + \pi 4^2 + 2\pi 4^2$
$96\pi + 16\pi + 32\pi$
$= 144\pi$

b.



$l = ?$
$10^2 + 2^2 = l^2$
$l = \sqrt{104}$
$l = 2\sqrt{26}$

$TA = LA + A_{\frac{1}{2}sphere}$
$= \pi rl + \frac{1}{2}(4\pi r^2)$
$= \pi 2(2\sqrt{26}) + 2\pi 4$
$TA = 4\pi\sqrt{26} + 8\pi$

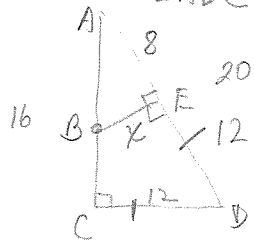
$V = V_{cone} + V_{\frac{1}{2}sphere}$
$\frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$\frac{40\pi}{3} + \frac{16\pi}{3}$
$V = \frac{56\pi}{3}$

11. Find the radius of the sphere inscribed in the right cone.

$V_{cone} = \frac{1}{3}Bh$
$\frac{1}{3}\pi 12^2(16)$
768π

$\frac{8}{16} = \frac{x}{12}$
$16x =$
$x =$

$\triangle AGE \sim \triangle ADC$



$h^2 + 12^2 = 20^2$
$h^2 = 256$
$h = 16$