Geometry (H) Name: KEY

Truth Tables & The Laws of Inference

Part I. Prove each law setting up a truth table. Remember to be a law the statement must be a tautology.

a. Law of Detachment: $[(p \to q) \wedge p] \to q$

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

yes!

b. Law of Disjunctive Inference (one version): $[(p \vee q) \wedge \sim p] \to q$

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

yes!

c. DeMorgan's Law (one version): $\sim (p \vee q) \leftrightarrow (\sim p \wedge \sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

yes!

Part II. Can the following be considered a law? Why or why not?
(Set up a truth table to find out)

$[(p \to q) \wedge q] \to p$

p	q	$(p \to q)$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

No; since one value turned out to be false, it is not a tautology and therefore, not a law

Part III. Draw a conclusion and then name the law that you are applying.

a. $f \wedge \sim g$

$\therefore f$ $\quad \therefore \sim g$

Law of Simplification

b. $\sim t \vee p$
$\sim p$

$\therefore \sim t$

L. of Disjunct. Inf.

c. $\sim b \to r$
$\sim r$

$\therefore b$

L. of Contrapositive Inf.

d. $p \to y$
$y \to \sim w$

$\therefore p \to \sim w$

L. of Syllogism

e. $\sim t \vee \sim p$

$\therefore$

f. $\sim z \to r$
$\sim z$

$\therefore r$

L. of Detachmt

g. $\sim(e \wedge \sim g)$

$\therefore \sim e \vee g$

DeMorgan's Law

h. $\sim k \vee \sim p$
k

$\therefore \sim p$

L. of Disjunct. Inf.

i. $v \to x$
$\sim x$

$\therefore \sim v$

L. of Contrapositive Inf.

j. $\sim j \wedge \sim s$

$\therefore \sim j$ $\quad \therefore \sim s$

L. of Simplification

k. $\sim t \to \sim p$
$\sim p \to h$

$\therefore \sim t \to h$

L. of Syllogism

l. $d \to \sim r$
d

$\therefore \sim r$

L. of Detachment

Part IV. Use the laws to write a flow proof for each of the following. Remember you proof is 2 part – statements & reasons. The reasons are either Given or one of the Laws.

a. $A \to B$
$\sim(C \wedge B)$
C

$\therefore \sim A$

$$\text{①}\ \sim(C \wedge B) \to \text{②}\ \sim C \vee \sim B \left.\begin{matrix} \\ \text{③}\ C \end{matrix}\right\} \to \text{④}\ \sim B \left.\begin{matrix} \\ \text{⑤}\ A \to B \end{matrix}\right\} \to \text{⑥}\ \sim A$$

① Given
② DeMorgan's Law
③ Given
④ L. of Disjunctive Inf.
⑤ Given
⑥ L. of Contrapositive Inf.

b. $\sim A \wedge C$
$A \vee W$
$R \to \sim W$
$\therefore \sim R$

$$\left.\begin{array}{l} (1)\ \sim A \wedge C \to (2)\ \sim A \\ (3)\ A \vee W \end{array}\right\} \to \left.\begin{array}{l} (4)\ W \\ (5)\ R \to \sim W \end{array}\right\} \to (6)\ \sim R$$

(1) Given
(2) L. of Simplification
(3) Given
(4) L. of Disjunctive Inf.
(5) Given
(6) L. of Contrapos. Inf.

c. $Q \to T$
$\sim(R \wedge \sim Q)$
R
$\therefore T$

$$\left.\begin{array}{l} (1)\ \sim(R \wedge \sim Q) \to (2)\ \sim R \vee Q \\ (3)\ R \end{array}\right\} \to \left.\begin{array}{l} (4)\ Q \\ (5)\ Q \to T \end{array}\right\} \to (6)\ T$$

(1) Given
(2) DeMorgan's Law
(3) Given
(4) Law of Disjunct. Inf.
(5) Given
(6) Law of Detachment.

Part V. Mixed review.

a. Write the following statement as a conditional. Determine if it is true or false.

A four-sided figure is a square.

F Conditional: If a figure is four-sided, then it is a square.

b. Write the converse, inverse and contrapositive of the conditional from part (a). Determine if each statement is true or false.

T Converse: If a figure is a square, then it is four sided.

T Inverse: If a figure is not four sided, then it is NOT a square.

F Contrapositive: If a figure is not a square, then it is NOT four sided.

c. Write the definition of an acute angle as a biconditional statement.

An angle is acute if and only if its measure is greater than 0° and less than 90°.

OR

An acute angle has a measure greater than 0° and less than 90°.

e. Determine if each compound statement is a tautology.

1. $[(p \vee q) \wedge \sim (p \wedge q)] \rightarrow (p \leftrightarrow q)$

p	q	$p \vee q$	$(p \wedge q)$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$	$(p \leftrightarrow q)$	[ ] → (
T	T	T	T	F	F	T	T
T	F	T	F	T	T	F	F
F	T	T	F	T	T	F	F
F	F	F	F	T	F	T	T

No, not a tautology!

2. $[(r \rightarrow s) \wedge (t \rightarrow \sim s)] \leftrightarrow (r \rightarrow t)$ (hint: There are 2 variables ... you will need 8 combinations of T's & F's)

SKIP

Part VI.

Represent using connectives and write a logical proof for the following:

a) Given:

If the radio is on and the television works, then there is electricity.
There is no electricity if the lights go out.
The lights go out.
The radio is on.

Let R represent: "The radio is on."
Let T represent: "The television works."
Let E represent: "There is electricity."
Let L represent: "The lights go out."

Using R, T, E, and L, prove: "The television does not work."

Given
$(R \wedge T) \to E$
$L \to \sim E$
L
R
Prove $\sim T$

① $L \to \sim E$, ② L → ③ $\sim E$
③ $\sim E$, ④ $(R \wedge T) \to E$ → ⑤ $\sim(R \wedge T)$ → ⑥ $\sim R \vee \sim T$
⑥, ⑦ R → ⑧ $\sim T$

① Given
② Given
③ L. Detachmt
④ Given
⑤ L. Contrapos.
⑥ DeMorgan's Law
⑦ Given
⑧ L. Disjunct. Inf.

$\boxed{\text{The TV does NOT work.}}$

b) Given
Either Demi went to college or she joined the army.
If she joined the army, then her hair was cut short.
If her hair was cut short, it does not cover her ears.
Demi's hair covers her ears.

Let J represent "Demi went to college."
Let A represent "She joined the army."
Let N represent "Her hair was cut short."
Let E represent "Her hair covers her ears."

a. Using J, A, N, E, and proper connectives, express each sentence in symbolic form.

b. Using laws of inference, show that Demi went to college.

Given
$J \vee A$
$A \to N$
$N \to \sim E$
E
$\therefore J$

① $N \to \sim E$, ② E → ③ $\sim N$
③ $\sim N$, ④ $A \to N$ → ⑤ $\sim A$
⑤ $\sim A$, ⑥ $J \vee A$ → ⑦ J

① Given
② Given
③ Contrapos. Inf.
④ Given
⑤ Contrapos Inf
⑥ Given
⑦ Disjunct. Inf.

c) Use a truth table to prove the Law of Disjunctive Inference.

$[(p \vee q) \wedge \sim q] \to p$

p	q	$(p \vee q)$	$\sim q$	$(p \vee q) \wedge \sim q$	$[\quad] \to p$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T