Name KEY

Advanced Algebra (H) - Linear Models

1. Handy Andy sells one-gallon cartons of milk (4 quarts) for $3.09 each and half-gallon cartons for $1.65 each. Assume that the number of cents you pay for a carton of milk varies linearly with the number of quarts the carton holds.

a. Write the particular equation expressing price (P) in terms of quarts (q).

(4, 3.09) (2, 1.65) $m = \frac{3.09 - 1.65}{4-2}$ $m = \$0.72$/quart

$1.65 = .72(2) + b$

$b = .21$

$P = 0.72q + .21$

b. If handy Andy sold 3-gallon cartons, what would your equation predict the price to be?

$P = .72(12) + .21 \rightarrow$ $P = \$8.85$

c. The actual prices for pint cartons ($\frac{1}{2}$ quart) and one-quart cartons are $ .57 and $ .99, respectively. Do these prices fit your mathematical model? If not, are they higher than predicted, or lower?

$q = \frac{1}{2}$

$P = .72(\frac{1}{2}) + .21$

$P = \$.57$ Yes, the pint fits.

$q = 1$

$P = .72 + .21$

$P = .93$

No, the one-qt. price is lower than the model.

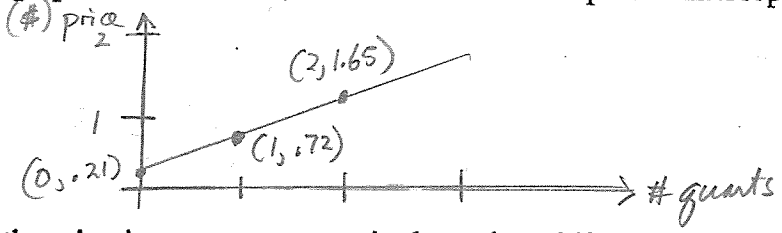
d. Suppose that you found cartons of milk marked at $3.45, but that there was nothing on the carton to tell what size it is. According to your model, how much would such a carton hold?

$3.45 = .72q + .21$

$q = 4.5$

It holds 4.5 quarts

e. Sketch the graph of the function, consistent with the slope and intercept in your equation.



f. What does the price-intercept represent in the real world?

$0.21 for zero quart of milk → this may represent the costs of overhead for Andy: rent, utilities, labor.

g. What are the units of the slope? What real-world quantity does this number represent?

The slope is 72¢ per quart.

2. Driving Home Problem: As you drive home from the football game, the number of kilometers you are away from home depends on the number of minutes you have been driving. Assume that the distance varies linearly with time. Suppose you are 11 km from home when you have been driving for 10 minutes, and 8 km from home when you have been driving for 15 minutes.

a. Write the particular equation expressing the number of kilometers you are from home (d) in terms of the number of minutes since you left the game (t).

(t, d) $(10, 11)$ $(15, 8)$

$m = \frac{11-8}{10-15}$ $\quad m = \frac{3}{-5}$

$11 = -\frac{3}{5}(10) + b$ $\quad 17 = b$

$d = -\frac{3}{5}t + 17$

or $d = -0.6t + 17$

b. Predict your distance from home after driving for 20 min., 25 min., and 30 min.

$t = 20$: $d = -\frac{3}{5}(20) + 17$ $\quad d = 5$ Km away

$t = 25$: $d = -\frac{3}{5}(25) + 17$ $\quad d = 2$ Km away

$t = 30$ $\quad d = -\frac{3}{5}(30) + 17$ $\quad d = -1$ Km

↓ arrived at home

c. When were you are 7 km from home, how many minutes have you been traveling?

$7 = -\frac{3}{5}t + 17$

$(-\frac{5}{3})(-10) = t$

$t = 16\frac{2}{3}$ minutes → 16 minutes 40 sec.

d. Find the distance-intercept. What does this number represent in the real world?

distance-intercept is 17

It means that the game was 17 Km from home.

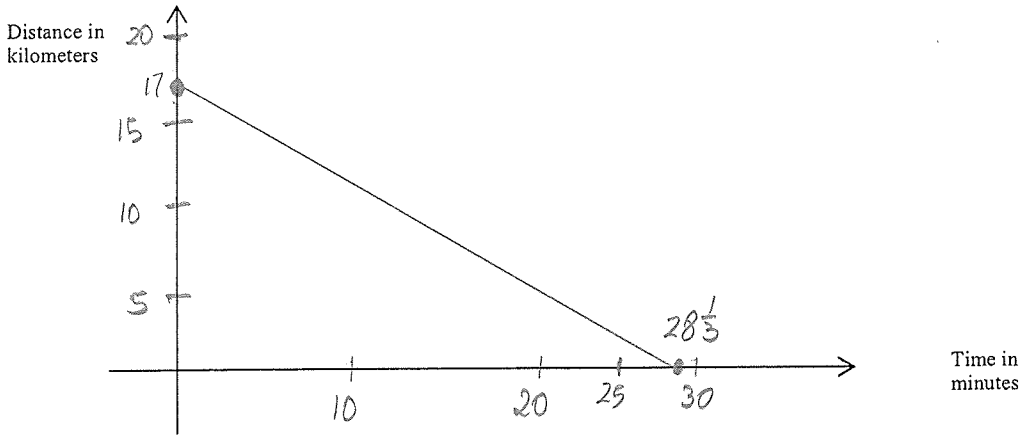
e. Find the time-intercept. What does this number represent in the real world?

$0 = -0.6t + 17$

$t = 28\frac{1}{3}$

It takes $28\frac{1}{3}$ or 28 min. 20 seconds to get home.

f. Plot the graph of this linear function. Use a suitable domain.



g. What is the slope? What does this number represent? What is the significance that the slope is negative?

slope is $-\frac{3}{5}$. It means that you drive 3 km per 5 minutes. The negative slope means that the distance away from home decreases over time.

3. **Bathtub Problem:** You pull out the plug from the bathtub. After 40 seconds, there are 13 gallons of water left in the tub. One minute after you pull the plug, there are 10 gallons left. Assume that the number of gallons varies linearly with the time since the plug was pulled.

a. Write the particular equation expressing the number of gallons (g) left in the tub in terms of the number of seconds (s) since you pulled the plug.

(s, g) $(40, 13)$ $(60, 10)$

$m = \frac{13-10}{40-60} = \frac{3}{-20}$

$g = sx + b$

$10 = \frac{3}{-20}(60) + b$

$10 = -9 + b$

$19 = b$

$g = -\frac{3}{20}s + 19$

b. How many gallons would be left after 20 seconds? 50 seconds?

$s = 20$ $g = -\frac{3}{20}(20) + 19 \rightarrow g = 16$ gallons

$s = 50$ $g = -\frac{3}{20}(50) + 19 \rightarrow g = 11\frac{1}{2}$ gallons

c. At what time will there be 7 gallons left in the tub?

$g = 7$

$7 = -\frac{3}{20}s + 19$

$-\frac{20}{3}(-12) = s$

$80 = s$ 80 seconds

d. Find the y-intercept (gallon-intercept). What does this number represent in the real world?

$g = 19$

It means that there were 19 gallons of water in the beginning.

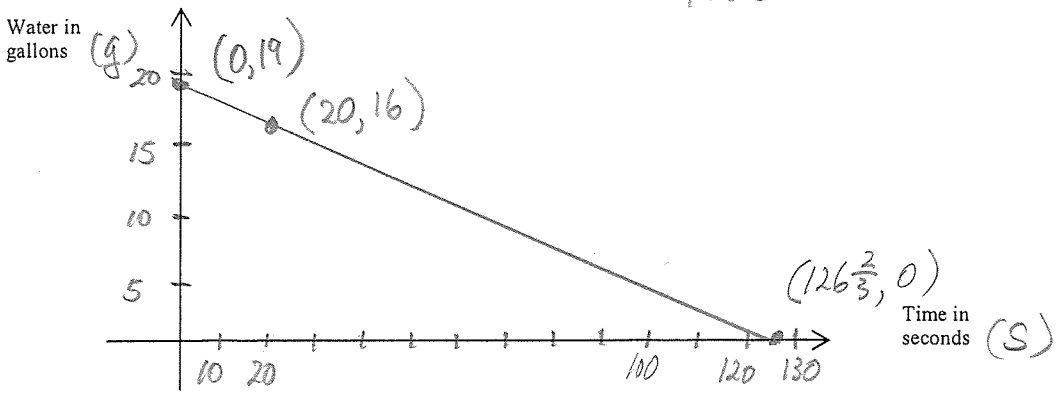
e. Find the x-intercept (time- intercept). What does this number represent in the real world?

$-19 = -\frac{3}{20}s$

$s = 126\frac{2}{3}$

It means that $126\frac{2}{3}$ seconds are needed to empty the tub.

f. Plot the graph of this linear function. Use a suitable domain.



g. What is the slope? What does this number represent?

slope $= -\frac{3}{20}$ It means 3 gallons are lost every 20 seconds.

4. **Taxi Problem:** To take a taxi in downtown St. Louis, it will cost you $3.00 to go a mile. ~~After~~ At 6 miles, it will cost $5.25. The cost varies linearly with the distance traveled.

a. Write the particular equation expressing cost (c) in terms of miles (d) traveled.

(d, c)
(1, 3)
(6, 5.25)

$m = \frac{5.25-3}{6-1}$

$= \frac{2.25}{5}$

$m = .45$

$3 = .45 + b$

$2.55 = b$

$C = .45d + 2.55$

b. How much will it cost you to travel 10 miles in a taxi?

$d = 10$

$C = .45(10) + 2.55$

$C = \$7.05$

c. How many miles can you travel if you only have $20 to spend?

$20 = .45d + 2.55$

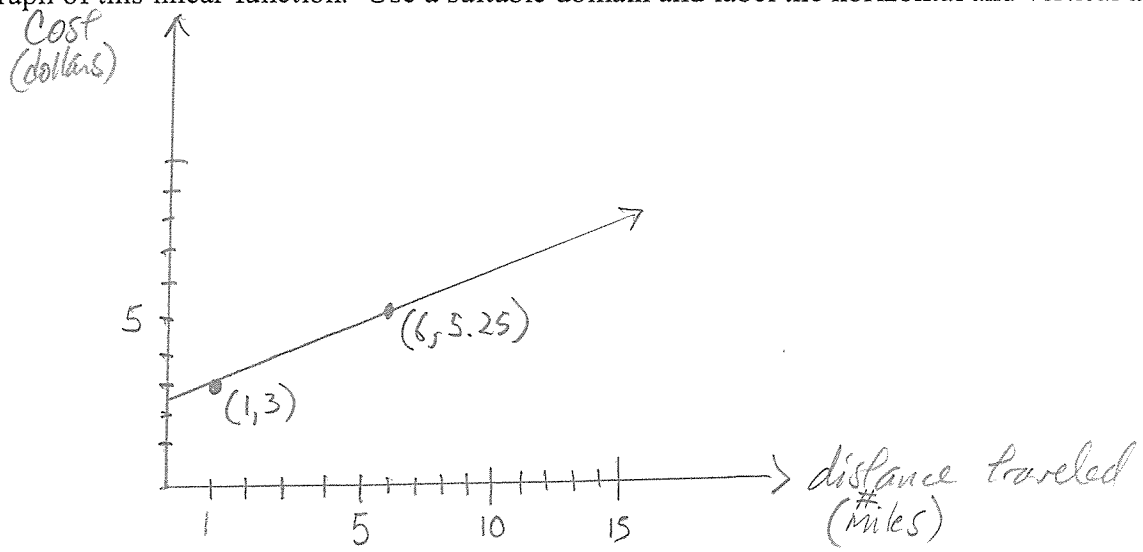
$d = 38\frac{7}{8}$ miles or 38 whole miles

d. Calculate the cost-intercept. What does this number represent in the real world?

cost-intercept = $2.55

This is the initial rate of the taxi before the taxi starts moving.

e. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.



g. What is the slope? What does this number represent?

Slope is $.45. This is the cost per mile after a flat fee of $2.55