

The four theorems on this sheet can be a step saver and a time saver! Get in the habit of using them when the situation arises and you'll save lots of time! You can cite these theorems by name. No need to write them out.

COMMON SEGMENT THEOREM:

Situation A

This is an explanation of the theorem. You can cite this theorem by using its name.

Explanation: If you add an **adjacent** segment to two separate segments AND by adding them, you join the two segments together, the two resulting segments will be congruent.

Given:

Diagram:

Prove:

Situation B

Explanation: If two segments overlap and the measure of the overlapping segment is subtracted from both segments, then the two resulting segments will be congruent.

Given: $\overline{AC} \cong \overline{BD}$

Diagram:



Conclude: $\overline{AB} \cong \overline{CD}$

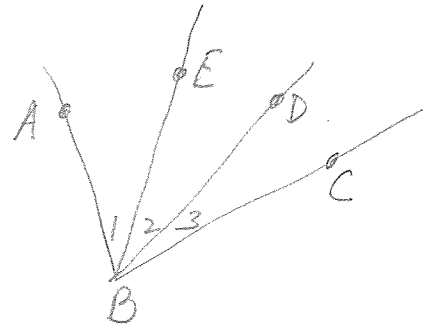
COMMON ANGLE THEOREM

Situation A

Given: $\angle ABD \cong \angle EBC$

Conclude: $\angle 1 \cong \angle 3$

Diagram:

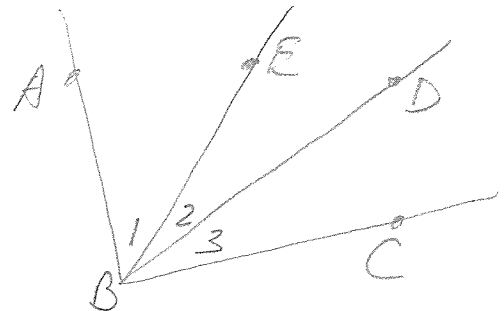


Situation B

Given: $\angle 1 \cong \angle 3$

Conclude: $\angle ABD \cong \angle EBC$

Diagram:



Midpoint Theorem

Given: X is midpt of \overline{AB}

Conclude: $AX = \frac{1}{2} AB$

$$XB = \frac{1}{2} AB$$

Diagram:



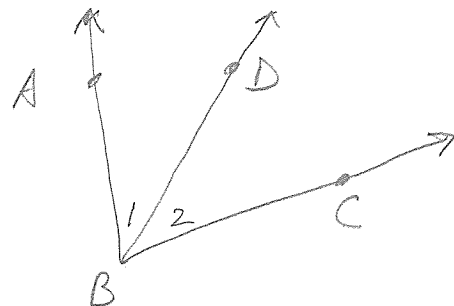
Angle bisector Theorem

Given: \overrightarrow{BD} bisector of $\angle ABC$

Conclude: $m\angle 1 = \frac{1}{2} m\angle ABC$

$$m\angle 2 = \frac{1}{2} m\angle ABC$$

Diagram:



Common Segment Thm

Add BC

(A)



Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

$$\begin{aligned} \textcircled{1} \overline{AB} \cong \overline{CD} &\rightarrow \textcircled{2} AB = CD \rightarrow \textcircled{3} AB + BC = CD + BC \\ &\quad \quad \quad \textcircled{4} \begin{aligned} AB + BC &= AC \\ BC + CD &= BD \end{aligned} \end{aligned} \left. \vphantom{\begin{aligned} \textcircled{1} \overline{AB} \cong \overline{CD} \\ \textcircled{2} AB = CD \\ \textcircled{3} AB + BC = CD + BC \\ \textcircled{4} \begin{aligned} AB + BC &= AC \\ BC + CD &= BD \end{aligned} \end{aligned}} \right\} \rightarrow \textcircled{5} AC = BD$$

① Given

② \cong segmts have = measures

③ Addition Prop.

④ Segment Add. Prop.

⑤ Substitution

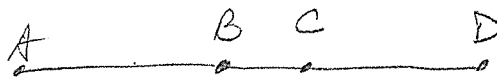
⑥ \cong segmts have = measures.

⑥ $\overline{AC} \cong \overline{BD}$

(B)

Given: $\overline{AC} \cong \overline{BD}$

Prove: $\overline{AB} \cong \overline{CD}$



Subtract BC

$$\begin{aligned} \textcircled{1} \overline{AC} \cong \overline{BD} &\rightarrow \textcircled{2} AC = BD \\ &\quad \quad \quad \textcircled{3} \begin{aligned} AB + BC &= AC \\ BC + CD &= BD \end{aligned} \end{aligned} \left. \vphantom{\begin{aligned} \textcircled{1} \overline{AC} \cong \overline{BD} \\ \textcircled{2} AC = BD \\ \textcircled{3} \begin{aligned} AB + BC &= AC \\ BC + CD &= BD \end{aligned} \end{aligned}} \right\} \rightarrow \textcircled{4} AB + BC = BC + CD \rightarrow \textcircled{5} AB = CD$$

⑥ $\overline{AB} \cong \overline{CD}$

① Given

② \cong segmts have = measures.

③ Segment Add. Post.

④ Substitution Prop.

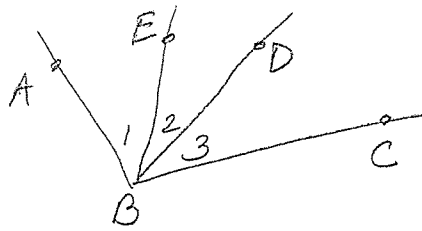
⑤ Subtraction Prop.

⑥ \cong segmts have = measures.

Common Angle Thm Proof "Subtracted from"

(A) Given: $\triangle ABD \cong \triangle EBC$

Prove: $\angle 1 \cong \angle 3$



$$\begin{aligned} \textcircled{1} \triangle ABD \cong \triangle EBC &\rightarrow \textcircled{2} m\angle ABD = m\angle EBC \\ \left. \begin{aligned} \textcircled{3} m\angle 1 + m\angle 2 &= m\angle ABD \\ m\angle 3 + m\angle 2 &= m\angle EBC \end{aligned} \right\} &\rightarrow \textcircled{4} m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 \end{aligned}$$

$$\textcircled{5} m\angle 1 = m\angle 3 \rightarrow \textcircled{6} \angle 1 \cong \angle 3$$

① Given

② $\cong \angle$ s have = measures

③ Angle addition Postulate

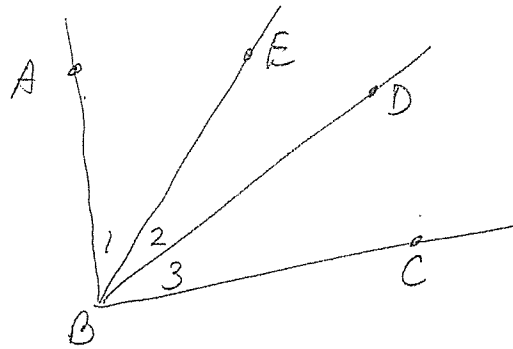
④ Substitution Prop

⑤ Subtraction Prop.

⑥ $\cong \angle$ s have = measures.

Proof

- (B) Given: $\angle 1 \cong \angle 3$
Prove: $\triangle ABD \cong \triangle EBC$



- (1) $\angle 1 \cong \angle 3 \rightarrow$ (2) $m\angle 1 = m\angle 3 \rightarrow$ (3) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$
(4) $m\angle 1 + m\angle 2 = m\angle ABD$
 $m\angle 2 + m\angle 3 = m\angle EBC$

- (5) $m\angle ABD = m\angle EBC \rightarrow$ (6) $\angle ABD \cong \angle EBC$

(1) Given

(2) $\cong \angle$ s have = measures.

(3) Addition Prop.

(4) Angle Add. Post.

(5) Substitution Prop.

(6) $\cong \angle$ s have equal measures.

Midpoint Thm