# Systems and Functions KEY

For Problems 1 through 26, let

$$f(x) = 3x + 11$$
$$g(x) = x^2 + x + 1$$

For Problems 17 through 26, you should realize that $f$ ("expression") means substitute "expression" for $x$. Evaluate, and simplify if possible.

17. $f(r)$
18. $g(n)$
19. $g(k)$
20. $f(j)$
21. $f(s + t)$
22. $g(4 - a)$
23. $g(f(x))$
24. $f(g(x))$
25. $f(f(x))$
26. $g(g(x))$

27. ***Cops and Robbers Problem*** Robin Banks robs a bank and drives off. A short time later he passes a truck stop at which police officer Willie Katchup is dining. Willie receives a call from his dispatcher, and takes off in pursuit of Robin.

Let $t$ = number of minutes that have elapsed since Robin passed the truck stop.
Let $f(t)$ = number of kilometers Robin has gone past the truck stop.
Let $g(t)$ = number of kilometers Willie has gone from the truck stop.

a. Robin's equation is $f(t) = 0.75t$. Find $f(12)$, $f(4)$, and $f(-8)$.
b. Willie's equation is $g(t) = 2(t - 5)$. Find $g(7)$ and $g(15)$.
c. By calculation, find the time and place Willie Katchup catches up with Robin Banks.
d. When did Willie leave the truck stop?
e. Sketch the graphs of functions $f$ and $g$ on the same set of axes, showing the point where they cross.
f. How fast were Robin and Willie going?

29. ***Efficient Car Problem*** A particular brand of car with the normal engine costs \$11,000 to purchase, and 22 cents a mile to drive. The same car with a fuel-injection engine costs \$11,300 to purchase, but only 20 cents a mile to operate.
a. Let $d$ be a variable equal to the number of miles you have driven the car, and $f(d)$ be the total number of dollars it costs to own the \$11,000 car. Write the particular equation for function $f$.
b. Calculate $f(1{,}000)$, $f(10{,}000)$, and $f(100{,}000)$.
c. Let $g(d)$ be the total number of dollars it costs to drive the \$11,300 car for $d$ miles. Write the particular equation for function $g$.
d. Calculate $g(1{,}000)$, $g(10{,}000)$, and $g(100{,}000)$.
e. How many miles would you have to drive to "break even?" That is, when does the total cost of owning the car with the normal engine equal the cost of owning the car with the fuel-injected engine?

(17) $f(r) = 3r + 11$

(18) $g(n) = n^2 + n + 1$

(19) $g(k) = k^2 + k + 1$

(20) $f(j) = 3j + 11$

(21) $f(s+t) = 3(s+t) + 11$
$= \boxed{3s + 3t + 11}$

(22) $g(4-a) = (4-a)^2 + (4-a) + 1$
$= 16 - 8a + a^2 + 4 - a + 1$
$= \boxed{21 - 9a + a^2}$

(23) $g(f(x)) = (3x+11)^2 + (3x+11) + 1$
$= 9x^2 + 66x + 121 + 3x + 12$
$= \boxed{9x^2 + 69x + 133}$

(24) $f(g(x)) = 3(x^2 + x + 1) + 11$
$= \boxed{3x^2 + 3x + 14}$

(25) $f(f(x)) = 3(3x+11) + 11$
$= \boxed{9x + 44}$

(26) $g(g(x)) =$
$(x^2+x+1)^2 + (x^2+x+1) + 1$
$x^4 + 2x^3 + 3x^2 + 2x + 1 + x^2 + x + 1 + 1$
$\boxed{x^4 + 2x^3 + 4x^2 + 3x + 3}$

#26 - Work for #26:

$$x^2+x+1$$
$$\underline{x^2+x+1}$$
$$x^2+x+1$$
$$x^3+x^2+x$$
$$\underline{x^4+x^3+x^2}$$
$$x^4+2x^3+3x^2+2x+1$$

**27.** *Cops and Robbers Problem*

**a.** $f(12) = \frac{3}{4}(12) = \underline{\underline{9}}$

$f(4) = \underline{\underline{3}}$

$f(-8) = \underline{\underline{-6}}$

**b.** $g(7) = 2(7 - 5) = \underline{\underline{4}}$

$g(15) = 2(15 - 5) = \underline{\underline{20}}$

**c.** At intersection point, $f(t) = g(t)$.

$\therefore \frac{3}{4}t = 2(t - 5)$

$t = 8$

$f(8) = g(8) = 6.$

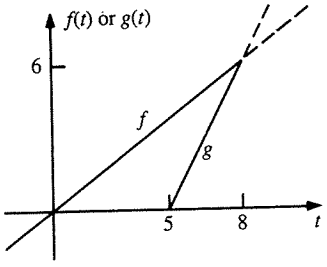
So Willie catches up after *8 minutes*, at *6 km* from the truck stop.

**d.** Willie left when $g(t) = 0$. So

$2(t - 5) = 0$

$t = \underline{\underline{5 \text{ min.}}}$

**e.** Graph.




**f.** Slopes are speeds in kilometers per *minute*.

Robin: $\frac{3}{4}(60) = \underline{\underline{45 \text{ km/h}}}$

Willie: $2(60) = \underline{\underline{120 \text{ km/h}}}$

**29.** *Efficient Car Problem*

**a.** $\underline{\underline{f(d) = 11{,}000 + 0.22d}}$

**b.** $f(1{,}000) = \underline{\underline{11{,}220}}$

$f(10{,}000) = \underline{\underline{13{,}200}}$

$f(100{,}000) = \underline{\underline{33{,}000}}$

**c.** $\underline{\underline{g(d) = 11{,}300 + 0.20d}}$

**d.** $g(1{,}000) = \underline{\underline{11{,}500}}$

$g(10{,}000) = \underline{\underline{13{,}300}}$

$g(100{,}000) = \underline{\underline{32{,}300}}$

**e.** $f(d) = g(d)$

$11{,}000 + 0.22d = 11{,}300 + 0.20d$

$0.02d = 300$

$d = 15{,}000$

$\underline{\underline{15{,}000 \text{ miles}}}$