Solve #1 algebraically. Then prove #2.

1. Find the measure of an angle and its supplement if the angle is 15° less than twice its supplement.

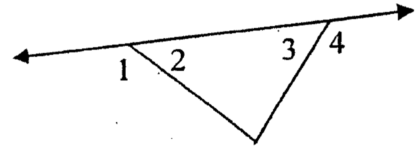
Let x = measure of $\measuredangle$
$180 - x$ = meas. of supplement
$x = 2(180 - x) - 15$
$x = 115$

$\{115°, 65°\}$

or Let x = meas. of supplement
$2x - 15$ = meas of $\measuredangle$
$2x - 15 + x = 180$
$3x = 195 \rightarrow x = 65$

$\{115°, 65°\}$

2. Given: $\angle 1$ supp $\angle 2$
$\angle 2 \cong \angle 3$
$\angle 4$ supp $\angle 3$



Prove: $\angle 1 \cong \angle 4$

optional step

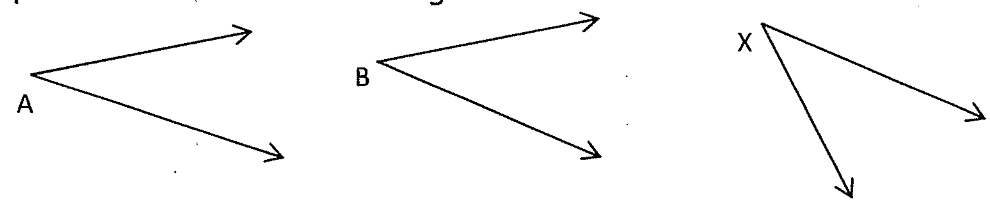
① $\measuredangle 1$ supp $\measuredangle 2$ → ③ $m\measuredangle 1 + m\measuredangle 2 = 180$
② $\measuredangle 4$ supp $\measuredangle 3$ → ④ $m\measuredangle 4 + m\measuredangle 3 = 180$
→ ⑤ $m\measuredangle 1 + m\measuredangle 2 = m\measuredangle 3 + m\measuredangle 4$
⑥ $\measuredangle 2 \cong \measuredangle 3$ → ⑦ $m\measuredangle 2 = m\measuredangle 3$
→ $m\measuredangle 1 + m\measuredangle 2 = m\measuredangle 2 + m\measuredangle 4$
→ ⑧ $m\measuredangle 1 = m\measuredangle 4$ → ⑨ $\measuredangle 1 \cong \measuredangle 4$

① Given.
② Given
③ Suppl. $\measuredangle$s: 2 $\measuredangle$s whose total measures = 180°.
④ Same as #3
⑤ Substitution prop.
⑥ Given
⑦ $\cong$ $\measuredangle$s have = measures
optional: Substitution prop
⑧ Subtraction prop.
⑨ $\cong$ $\measuredangle$s have = measures.

Congratulations! You've just proven a theorem!
This is the Congruent Supplements Theorem which states that if 2 angles supplement $\cong$ angles or the same angle, then they are $\cong$ to each other.
(short version: Supplements of $\cong$ $\measuredangle$s are $\cong$.)

(3)

<u>Congruent complements theorem:</u> If two angles complement the same angle or are complementary to congruent angles, then the two angles are congruent. ➔ ∠s that complement the same ∠s are congruent.



Given: ∠A complement ∠x
∠B complement ∠x.

Prove: ∠A ≅ ∠B

(1) $\angle A$ compl. $\angle X$ → (2) $m\angle A + m\angle X = 90$
$\angle B$ compl. $\angle X$ → $m\angle B + m\angle X = 90$ } → (3) $m\angle A + m\angle X = m\angle B + m\angle X$

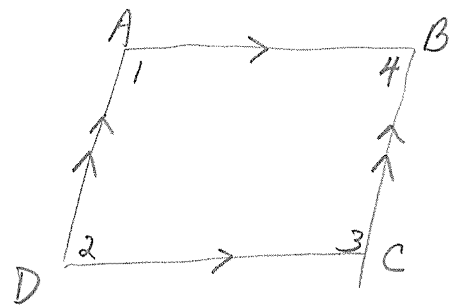
(4) $m\angle A = m\angle B$ → (5) $\angle A \cong \angle B$

(1) Given
(2) Compl. ∠s are 2 ∠s whose sum is 90°.
(3) Substitution Prop.
(4) Subtraction Prop.
(5) ≅ ∠s have = measures.

4. Draw a four-sided figure $ABCD$ with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$. ← Given

a. Prove that $\angle A \cong \angle C$.

b. Is $\angle B \cong \angle D$? Explain in a paragraph

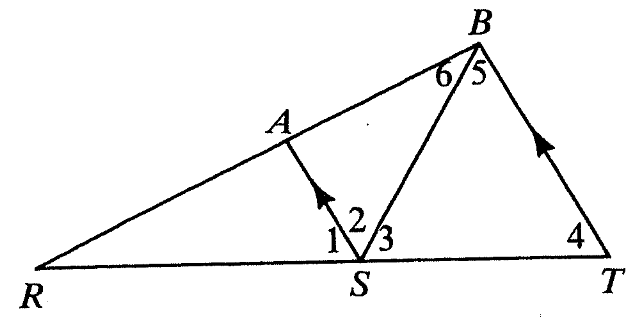


a) ① $\overline{AB} \parallel \overline{DC}$ → ② $\angle 3$ & $\angle 4$ supplements
③ $\overline{AD} \parallel \overline{BC}$ → ④ $\angle 1$ & $\angle 4$ supplements
→ ⑤ $\angle 1 \cong \angle 3$

① Given
② 2 ∥ lines → same-side interior ∠s supplementary
③ Given
④ Same as #2
⑤ Angles that supplement the same ∠ are ≅.

b) Since it's given that $\overline{AD} \parallel \overline{BC}$, ∠1 supplements ∠4, b'se if 2 lines ∥ → same-side interior ∠s are supplementary. It's also given that $\overline{AB} \parallel \overline{DC}$ which means ∠1 supplements ∠2 b'se if 2 lines ∥ → same-side interior ∠s supplementary. Consequently, ∠2 ≅ ∠4 because supplements of the same ∠ are ≅ to each other. Therefore, ∠B ≅ ∠D.

5. Given: $\overline{AS} \parallel \overline{BT}$;
$m\angle 4 = m\angle 5$
Prove: $\overrightarrow{SA}$ bisects $\angle BSR$.
(Plan: Show $\angle 1 \cong \angle 2$)



$$\left.\begin{array}{l} ① \ m\angle 4 = m\angle 5 \rightarrow ② \ \angle 4 \cong \angle 5 \\ ③ \ \overline{AS} \parallel \overline{BT} \rightarrow ④ \ \angle 4 \cong \angle 1 \end{array}\right\} \rightarrow \left.\begin{array}{l} ⑤ \ \angle 1 \cong \angle 5 \\ ⑥ \ \angle 5 \cong \angle 2 \end{array}\right\} \rightarrow ⑦ \ \angle 1 \cong \angle 2 \rightarrow ⑧ \ \overrightarrow{SA} \text{ bisects } \angle BSR.$$

① Given
② $\cong$ $\angle$s have = measures
③ Given
④ 2 $\parallel$ lines $\rightarrow$ corresponding $\angle$s $\cong$.
⑤ Transitive Prop
⑥ 2 $\parallel$ lines $\rightarrow$ alt. int. $\angle$s $\cong$.
⑦ Transitive Prop
⑧ An $\angle$ bisector $\div$ an $\angle$ into 2 $\cong$ parts.

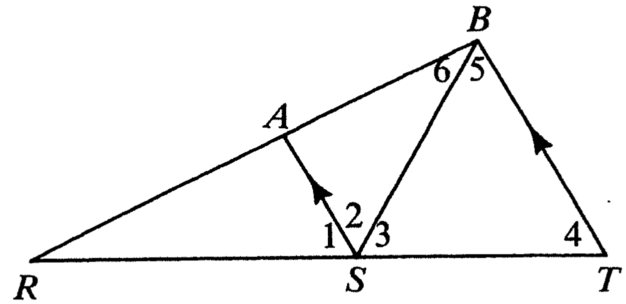
6. Given: $\overline{AS} \parallel \overline{BT}$;

$m\angle 4 = m\angle 5$;

$\overrightarrow{SB}$ bisects $\angle AST$.

Find the measure of $\angle 1$.

(Not a proof problem)



Since $\overline{AS} \parallel \overline{BT} \rightarrow \angle 4 \cong \angle 1$ (corresponding $\angle$s of $\parallel$ lines)

$\rightarrow \angle 5 \cong \angle 2$ (alt. int. $\angle$s of $\parallel$ lines)

Since $m\angle 4 = m\angle 5 \rightarrow \left.\begin{matrix}\angle 4 \cong \angle 5 \\ \angle 5 \cong \angle 2\end{matrix}\right\} \rightarrow \angle 4 \cong \angle 2$ (transitive prop.)

$\rightarrow \angle 3 \cong \angle 4$ (transitive prop)

Since $\overrightarrow{SB}$ bisects $\angle AST \rightarrow \angle 2 \cong \angle 3$

Since $\angle 3 \cong \angle 4$ and $\angle 4 \cong \angle 5 \rightarrow \angle 3 \cong \angle 4 \cong \angle 5$ and

$m\angle 3 = m\angle 4 = m\angle 5$. Since measure of 3 $\angle$s of $\triangle = 180°$,

$m\angle 3 = 60°$ and $m\angle 4 = 60$. Since $\angle 4 \cong \angle 1$, $m\angle 1 = 60°$.