

Problem Solving

1. A rectangle has an area of 20 square inches and a perimeter of 18 inches. Find its dimensions.

let L = length
 W = width

$$\begin{aligned} LW &= 20 \\ 2L + 2W &= 18 \end{aligned}$$

$$\begin{aligned} 2L + 2W &= 18 \\ L + W &= 9 \\ L &= 9 - W \\ W(9 - W) &= 20 \\ 9W - W^2 &= 20 \\ 0 &= W^2 - 9W + 20 \end{aligned}$$

$$\begin{aligned} (W-4)(W-5) \\ W=4, W=5 \\ \downarrow \quad \downarrow \\ L=5 \quad L=4 \end{aligned}$$

Dimensions
length = 5 inches
width = 4 inches

2. The product of two positive numbers is 15 and the sum of their squares is 34. Find the numbers.

let x = one #
 y = another #

$$\begin{aligned} xy &= 15 \\ x^2 + y^2 &= 34 \end{aligned}$$

$$\begin{aligned} x &= \frac{15}{y} \\ \left(\frac{15}{y}\right)^2 + y^2 &= 34 \\ \frac{225}{y^2} + y^2 &= 34 \\ 225 + y^4 &= 34y^2 \end{aligned}$$

$$\begin{aligned} y^4 - 34y^2 + 225 &= 0 \\ (y^2 - 9)(y^2 - 25) &= 0 \\ y = \pm 3 \quad y = \pm 5 \\ \downarrow \text{omit } -3 \quad \downarrow \text{omit } -5 \\ x = 5 \quad x = 3 \end{aligned}$$

The 2 #s are
3 & 5

3. Find the dimensions of a rectangle if the area is 12 cm² and its length is 2 cm less than twice its width.

let L = length
 W = width

$$\begin{aligned} LW &= 12 \\ L &= 2W - 2 \end{aligned}$$

dimensions are

$$\begin{aligned} 3 \text{ cm} \ \& \ 4 \text{ cm} \\ \downarrow \\ 2(3) - 2 &= 4 \checkmark \end{aligned}$$

$$\begin{aligned} W(2W - 2) &= 12 \\ 2W^2 - 2W - 12 &= 0 \\ W^2 - W - 6 &= 0 \\ (W - 3)(W + 2) &= 0 \\ W = 3, W = -2 \\ &\text{extraneous} \end{aligned}$$

4. The area of a rectangle is $\sqrt{3}$ square meters and the length of the diagonal is 2 meters. Find the dimensions.

let L = length
 W = width

$$\begin{aligned} LW &= \sqrt{3} \\ L^2 + W^2 &= 4 \\ L^2 &= 4 - W^2 \\ L &= \sqrt{4 - W^2} \end{aligned}$$

$$\begin{aligned} W \sqrt{4 - W^2} &= \sqrt{3} \\ W^2(4 - W^2) &= 3 \\ 4W^2 - W^4 &= 3 \\ W^4 - 4W^2 + 3 &= 0 \\ (W^2 - 3)(W^2 - 1) &= 0 \\ W = \pm\sqrt{3} \quad W = \pm 1 \end{aligned}$$

Dimensions: $\sqrt{3}$ meters
1 meter

$$\begin{aligned} \text{ck: } (\sqrt{3})^2 + 1^2 &\stackrel{?}{=} 4 \\ 3 + 1 &= 4 \\ 4 &= 4 \end{aligned}$$