Advanced Algebra (H) Date _____

Quadratics 2.1 Writing equations of Transformations of Quadratic Functions

Question: Is the order of the transformations important?

Example 1: The function $f(x)$ $y=x^2$ undergoes a vertical stretch by a factor of 2, then a reflection across the x-axis, and then a translation 3 units down. Write a rule for g, the new function, and identify the vertex. Check with your calculator.

① $h(x) = 2x^2$

② $h(x) = -2x^2$

③ $g(x) = -2x^2 - 3$ $\Rightarrow$ $g(x) = -2(x-0)^2 - 3$

vertex $(0, -3)$

Example 2: The function $f(x)$ $y = x^2$ is translated 3 units down, then reflected across the x-axis, followed by a vertical stretch by a factor of 2. Write a rule for g, the new function, and identify the vertex. Check with your calculator.

① $h(x) = x^2 - 3$

② $h(x) = -(x^2 - 3)$
$= -x^2 + 3$

③ $h(x) = 2(-x^2 + 3)$

④ $g(x) = -2x^2 + 6$
$g(x) = -2(x-0)^2 + 6$

vertex $(0, 6)$

* See graphs →

Example 3: Let the graph of g be a translation 3 units right and 2 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^2 - 5x$. Write a rule for g.

① $h(x) = f(x-3) + 2$ ← shift f up 2x
↑ Input into f

Working function →

② $h(x) = (x-3)^2 - 5(x-3) + 2$ plug $(x-3)$ into $f(x)$
$= x^2 - 6x + 9 - 5x + 15 + 2$ } Simplify

③ $h(x) = x^2 - 11x + 26$

④ Reflection : $h(-x) = (-x)^2 - 11(-x) + 26$

⑤ final : $g(x) = x^2 + 11x + 26$

PRACTICE:

1. Let the graph of g by a vertical shrink by a factor of ½ followed by a translation 2 units up of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex.

$h(x) = \frac{1}{2}x^2$

$g(x) = \frac{1}{2}x^2 + 2$

vertex $(0, 2)$

2. Carry out problem #1 but reverse the order of transformations. Write a rule for g and identify the vertex.

$h(x) = x^2 + 2$

$g(x) = \frac{1}{2}(x^2 + 2)$ → $g(x) = \frac{1}{2}x^2 + 1$

vertex $(0, 1)$

3. Let the graph of g be a translation 4 units left followed by a vertical shrink by a factor of 1/3 of the graph of $f(x) = x^2 + x$. Write a rule for g and identify the vertex.

$h(x) = x^2 + x$

$= (x+4)^2 + (x+4)$

$= x^2 + 8x + 16 + x + 4$

$= x^2 + 9x + 20$

$-3.\frac{3}{2}$

$g(x) = \frac{1}{3}(x^2 + 9x + 20)$

$g(x) = \frac{1}{3}x^2 + 3x + \frac{20}{3}$

vertex: $x = \frac{-3}{2(\frac{1}{3})} = -\frac{9}{2}$

$y = (\frac{81}{4} + \frac{-81}{2} + 20)\frac{1}{3}$

$\frac{1}{3}(-\frac{81}{4} + \frac{80}{4}) \rightarrow \frac{1}{3}(-\frac{1}{4})$

vertex $(-\frac{9}{2}, -\frac{1}{12})$

4. Write a function g based on the transformations of $f(x) = 2x^2 + 6x$: a translation 6 units down followed by a reflection in the x-axis. Write a rule for g and identify the vertex.

$h(x) = 2x^2 + 6x - 6$

$h(x) = -(2x^2 + 6x - 6)$

$g(x) = -2x^2 - 6x + 6$

vertex $(-\frac{3}{2}, \frac{21}{2})$

$x = \frac{6}{-4} = -\frac{3}{2}$

$y = -2(\frac{9}{4}) - 6(-\frac{3}{2}) + 6$

$= -\frac{9}{2} + 9 + 6 \rightarrow 10\frac{1}{2} = \frac{21}{2}$

5. Write a function g based on the transformations of $f(x) = 2x^2 + 6x$: a reflection in the y-axis followed by a translation 4 units right.

① $h(-x) = 2(-x)^2 + 6(-x)$

$= 2x^2 - 6x$

② $h(x-4) = 2(x-4)^2 - 6(x-4)$

$= 2(x^2 - 8x + 16) - 6x + 24$

$= 2x^2 - 16x + 32 - 6x + 24$

$g(x) = 2x^2 - 22x + 56$