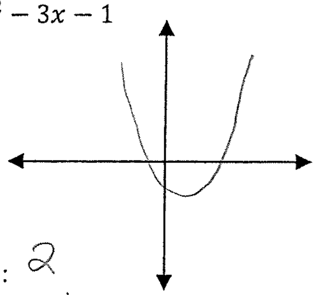
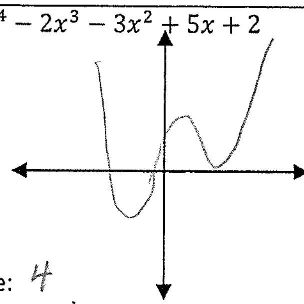
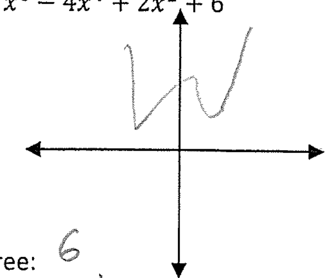
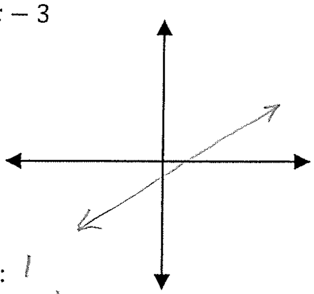
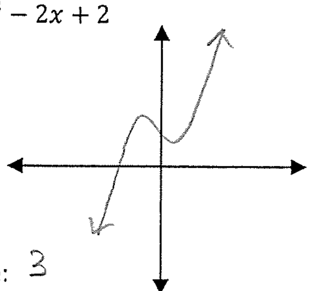
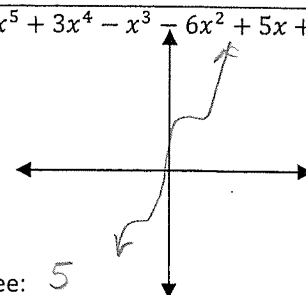
Graphing Polynomial Functions

Objective: To graph a polynomial function and to determine an equation for a polynomial function.

Name: KEY

Question: What can the degree and leading coefficient of a polynomial tell you about the graph?

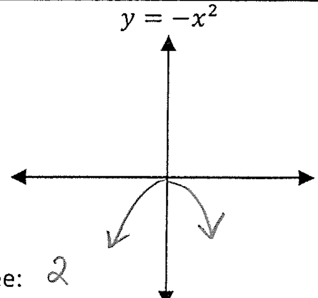
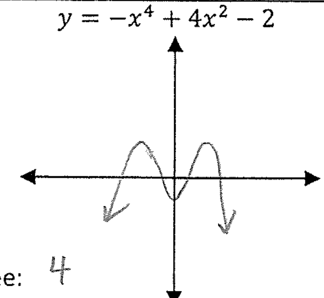
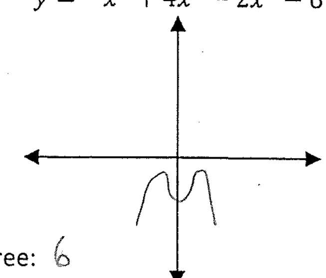
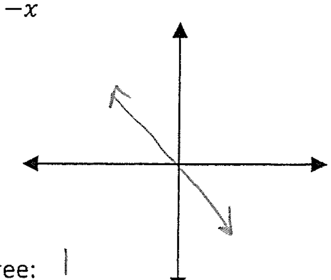
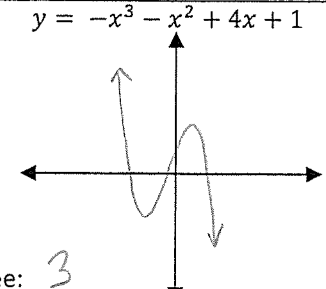
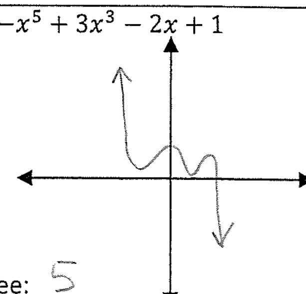
Part I. Use a graphing calculator, make a rough sketch of each polynomial, then give the degree and sign of the leading coefficient.

$y = x^2 - 3x - 1$  Degree: 2 Sign of LC: +	$y = x^4 - 2x^3 - 3x^2 + 5x + 2$  Degree: 4 Sign of LC: +	$y = x^6 - 4x^4 + 2x^2 + 6$  Degree: 6 Sign of LC: +
$y = 2x - 3$  Degree: 1 Sign of LC: +	$y = x^3 - 2x + 2$  Degree: 3 Sign of LC: +	$y = x^5 + 3x^4 - x^3 - 6x^2 + 5x + 2$  Degree: 5 Sign of LC: +

1. Describe the end behavior of the graph of a polynomial with an **EVEN DEGREE** and **POSITIVE LEADING COEFFICIENT**.
As x approaches negative infinity, y $\rightarrow +\infty$.
As x approaches positive infinity, y $\rightarrow +\infty$.

2. Describe the end behavior of the graph of a polynomial with an **ODD DEGREE** and **POSITIVE LEADING COEFFICIENT**.
As x approaches negative infinity, y $\rightarrow -\infty$.
As x approaches positive infinity, y $\rightarrow +\infty$.

Let's do it again!

$y = -x^2$  Degree: 2 Sign of LC: −	$y = -x^4 + 4x^2 - 2$  Degree: 4 Sign of LC: −	$y = -x^6 + 4x^4 - 2x^2 - 6$  Degree: 6 Sign of LC: −
$y = -x$  Degree: 1 Sign of LC: −	$y = -x^3 - x^2 + 4x + 1$  Degree: 3 Sign of LC: −	$y = -x^5 + 3x^3 - 2x + 1$  Degree: 5 Sign of LC: −

3. Describe the end behavior of the graph of a polynomial with an **EVEN DEGREE** and **NEGATIVE LEADING COEFFICIENT**.
As x approaches negative infinity, y $\rightarrow -\infty$.
As x approaches positive infinity, y $\rightarrow -\infty$.

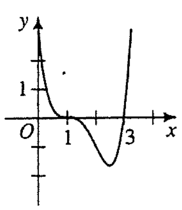
4. Describe the end behavior of the graph of a polynomial with an **ODD DEGREE** and **NEGATIVE LEADING COEFFICIENT**.
As x approaches negative infinity, y $\rightarrow +\infty$.
As x approaches positive infinity, y $\rightarrow -\infty$.

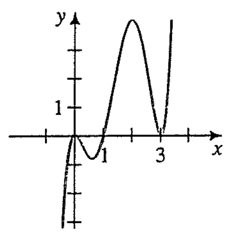
Part II. Effect of a First, Second and Cubed Factor:

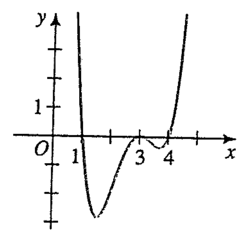
*If a polynomial has a first factor such as $(x-c)^1$ then the graph *passes through* the x-axis at x=c.
think linear

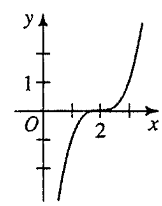
**If a polynomial has a squared factor such as $(x-c)^2$ then the graph is *tangent to* the x-axis at x=c; x=c is double root.
like a parabola

***If a polynomial has a cubed factor such as $(x-c)^3$ then the graph *flattens out* around x=c and crosses the x-axis at this point; x=c is triple root. called pt of inflection; the large the exponent, the flatter

1.

Quartic
$y=(x-1)^3(x-3)$

2.

Quintic (5th degree)
$y=x^2(x-1)(x-3)^2$

3.

Quartic
$y=(x-1)(x-3)^2(x-4)$

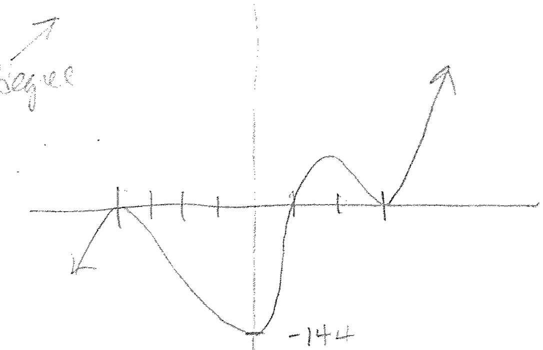
4.

Cubic
$y=(x-2)^3$

Notice turning points - where increasing interval or decreasing interval takes place

Part III. Sketch and label all intercepts.

1. $f(x)=(x+4)^2(x-1)(x-3)^2$ 5th degree

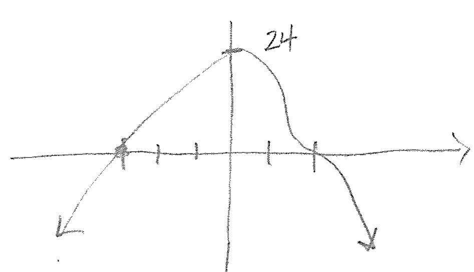
tangent to -4 and 3
passes thru 1



y int
x=0
f(x) = -144

2. $f(x)=-(x+3)(x-2)^3$ neg 4th degree

passes thru -3
flattens at 2



y int
x = 0
f(x) = -(3)(-8) = 24

3. $f(x)=-2x^3-x^2+14x-3$

(0, -3)

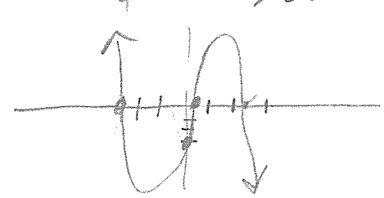
b: ±1, ±3
a: ±1, ±2
$\frac{b}{a}$: ±1, ±$\frac{1}{2}$, ±3, ±$\frac{3}{2}$

P(-3) = 0
factor is (x+3)
y-int = -3

-3	-2	-1	14	-3
		6	-15	3
	-2	5	-1	0

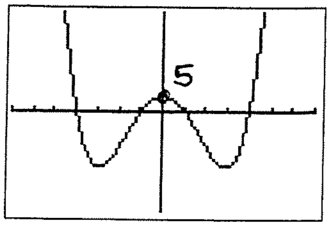
$-2x^2+5x-1$
a = -2, b = 5, c = -1

$x=\frac{-5\pm\sqrt{25-4(-2)(-1)}}{-4}$
$=\frac{-5\pm\sqrt{17}}{-4}$ → 0.22 → 2.28



Part IV. Write a factored form polynomial function f(x) of least degree.

4.



y-int = 5
even degree
+ LC

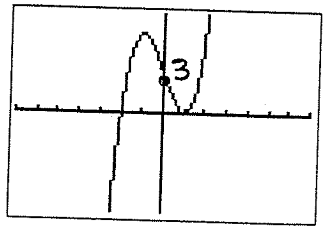
$P(x) = K(x+4)(x+1)(x-1)(x-4)$

$5 = K(16)$

$\frac{5}{16} = K$

$P(x) = \frac{5}{16}(x+4)(x+1)(x-1)(x-4)$

5.



odd
+ LC
y-int = 3

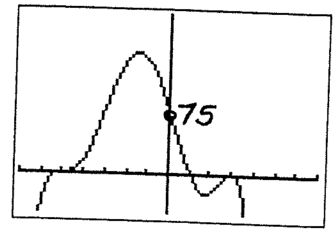
$P(x) = K(x+2)(x-1)^2$

$3 = K(2)$

$\frac{3}{2} = K$

$P(x) = \frac{3}{2}(x+2)(x-1)^2$

6. class



even
− LC
y-int = 75

$P(x) = K(x+5)^3(x-1)(x-3)^2$

$75 = K(125)(-9)$

$\frac{3}{-9(5)} = K \rightarrow \frac{1}{-15}$

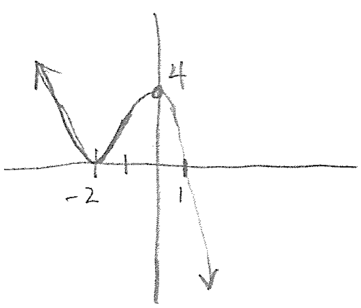
$P(x) = -\frac{1}{15}(x+5)^3(x-1)(x-3)^2$

Part V. Without a calculator, sketch the graph of each polynomial function using the information provided.

7. A polynomial with a negative leading coefficient and zeros of x = -2 (multiplicity 2) and x = 1.

$P(x) = -(x+2)^2(x-1)$

↑ ↓

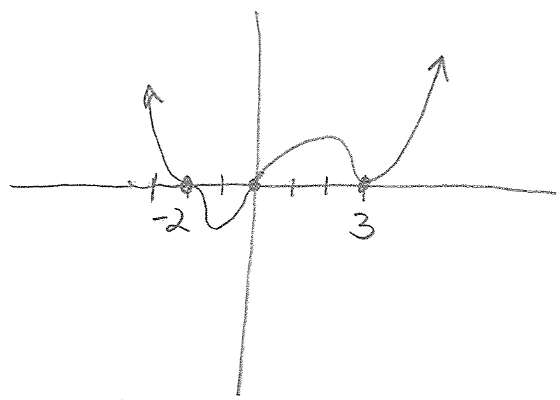


class

8. A polynomial with a positive leading coefficient and zeros of x = -2 (multiplicity 3), x = 0, and x = 3 (multiplicity 2).

$P(x) = (x+2)^3(x)(x-3)^2$

even
↑ ↑



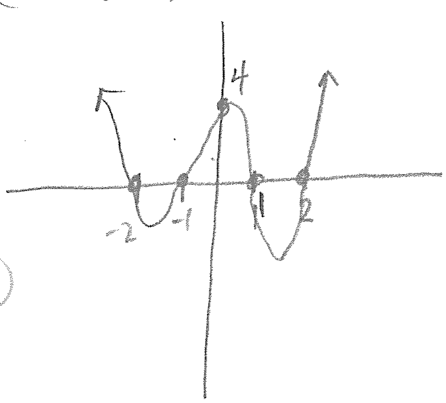
Part VI. Sketch and label all intercepts.

9. $y = x^4 - 5x^2 + 4$

↑ ↑

$0 = (x^2-4)(x^2-1)$

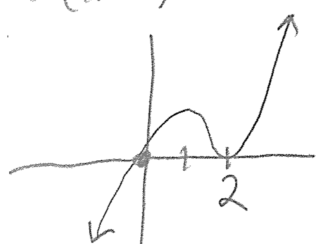
$(x+2)(x-2)(x+1)(x-1)$



10. $y = x^3 - 4x^2 + 4x$

$0 = x(x^2-4x+4)$

$x(x-2)^2$



(do w/ class)

11. $y = 2x^3 + 8x^2 - 3x - 12$

$0 = 2x^3 + 8x^2 - (3x+12)$

$2x^2(x+4) - 3(x+4)$

$0 = (2x^2-3)(x+4)$

$2x^2 = 3$

$x^2 = \frac{3}{2}$

don't rationalize
get decimal value

y-int = -12 $\quad x = \pm\frac{\sqrt{3}}{\sqrt{2}} \approx \pm 1.225 \quad x = -4$