Geometry Honors Name Date: KEY

10.5 Perimeter and Area on the Coordinate Plane

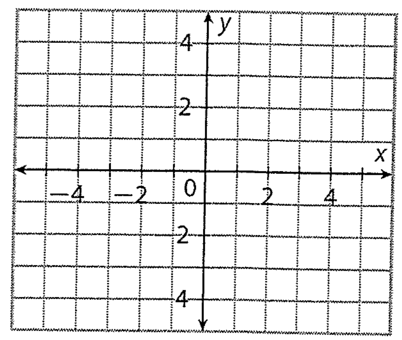
How do we find the perimeter and area of composite polygons in the coordinate plane?

Finding Perimeters of Figures on the Coordinate Plane

Recall that the perimeter of a polygon is the sum of the lengths of the polygon's sides. You can use the Distance Formula to find perimeters of polygons in a coordinate plane.

Follow these steps to find the perimeter of a pentagon with vertices $A(-1, 4)$, $B(4, 4)$, $C(3, -2)$, $D(-1, -4)$, and $E(-4, 1)$. Round to the nearest tenth.

(A) Plot the points. Then use a straightedge to draw the pentagon that is determined by the points.



(B) Are there any sides for which you do not need to use the Distance Formula? Explain, and give their length(s). $\overline{AB}$

$AB = 5$; $\overline{AB}$ is horizontal so we can count the boxes.

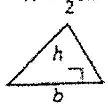
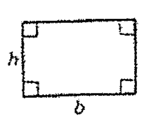
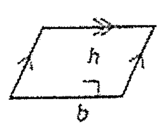
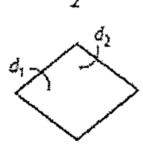
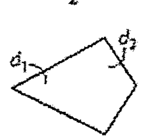
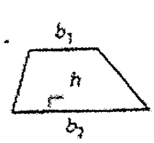
(C) Use the Distance Formula to find the remaining side lengths. Round your answers to the nearest tenth.

$BC = \sqrt{37}$ $CD = \sqrt{20} = 2\sqrt{5}$ $DE = \sqrt{34}$ $AE = \sqrt{18}$ $3\sqrt{2}$

(D) Find the sum of the side lengths.

Perimeter $= 5 + \sqrt{37} + 2\sqrt{5} + \sqrt{34} + 3\sqrt{2}$
≈ 25.6 units

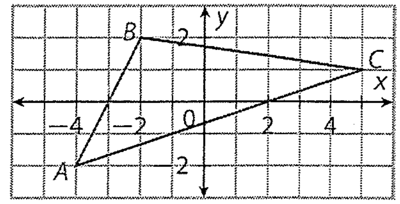
You can use area formulas together with the Distance Formula to determine areas of figures such as triangles, rectangles and parallelograms.

triangle $A = \frac{1}{2}bh$ 	rectangle $A = bh$ 	parallelogram $A = bh$ 
rhombus $A = \frac{1}{2}d_1 d_2$ 	kite $A = \frac{1}{2}d_1 d_2$ 	trapezoid $A = \frac{1}{2}(b_1 + b_2)h$ 

Example 1 **Find the area of each figure.**

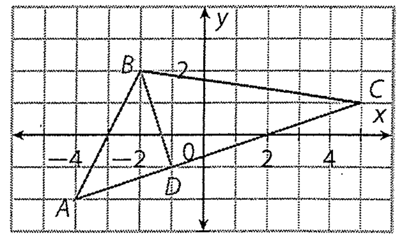
Ⓐ **Step 1** Find the coordinates of the vertices of $\triangle ABC$.

$A(-4,-2)$ $B(-2,2)$ $C(5,1)$



Step 2 Choose a base for which you can easily find the height of the triangle.

Step 3 Determine the area of $\triangle ABC$.



Base $\overline{AC}$. $D(-1,-1)$

verify $\overline{AC} \perp \overline{BD}$?

$m_{\overline{AC}} = \frac{1}{3}$

$m_{\overline{BD}} = -3$

$\therefore \overline{AC} \perp \overline{BD}$

$AC = \sqrt{90}$

$= 3\sqrt{10}$

$BD = \sqrt{10}$

$A = \frac{1}{2}bh$

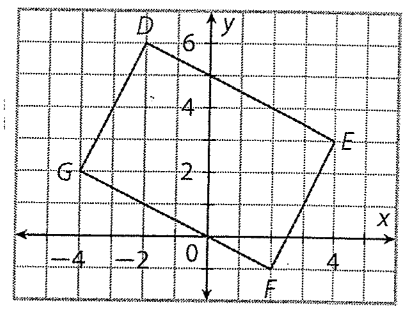
$= \frac{1}{2}\, 3\sqrt{10}(\sqrt{10})$

$\boxed{A = 15 \text{ sq. units}}$

Ⓑ **Step 1** Find the coordinates of the vertices of *DEFG*.

Step 2 *DEFG* appears to be a rectangle. Use slopes to check that

Step 3 Find the area of *DEFG*.



$D(-2,6), E(4,3), F(2,-1), G(-4,2)$

$m_{\overline{DE}} = \frac{6-3}{-6} = -\frac{1}{2}$

$m_{\overline{GF}} = \frac{-1-2}{2+4} = \frac{-3}{6} = -\frac{1}{2}$

$m_{\overline{EF}} = \frac{3+1}{4-2} = 2$

$m_{\overline{DG}} = 2$

$\therefore$ DEFG is a rectangle.

$A = LW$

$DG = \sqrt{20}$

$= 2\sqrt{5}$

$FG = \sqrt{45}$

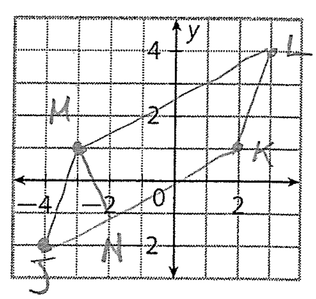
$= 3\sqrt{5}$

$A = 2\sqrt{5}(3\sqrt{5})$

$A = \boxed{30 \text{ sq. units}}$

Your Turn

4. Find the area of quadrilateral *JKLM* with vertices $J(-4,-2)$, $K(2,1)$, $L(3,4)$, $M(-3,1)$.



Verify JKLM is a ▱

$m_{\overline{ML}} = \frac{1}{2}$

$m_{\overline{JK}} = \frac{1}{2}$

$\overline{ML} \parallel \overline{JK}$

$m_{\overline{MJ}} = 3$

$m_{\overline{LK}} = 3$

$\overline{MJ} \parallel \overline{LK}$

Verify $\overline{MN} \perp \overline{JK}$

$m_{\overline{MN}} = -2$

$m_{\overline{JK}} = \frac{1}{2}$ ✓

$MN = \sqrt{5}$

$JK = \sqrt{45} = 3\sqrt{5}$

$A = bh$

$= 3\sqrt{5}(\sqrt{5})$

$\boxed{A = 15 \text{ sq units}}$

Finding Areas of Composite Figures

A composite figure is made up of simple shapes, such as triangles, rectangles, and parallelograms. To find the area of a composite figure, find the areas of the simple shapes and then use the Area Addition Postulate. You can use the Area Addition Postulate to find the area of a composite figure.

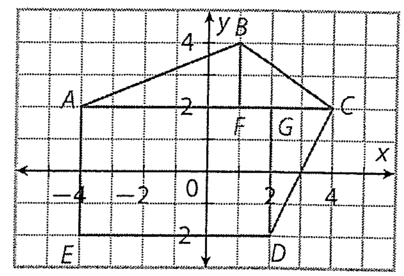
Area Addition Postulate

The area of a region is equal to the sum of the areas of its nonoverlapping parts.

Example 2 **Find the area of each figure.**

(A) Possible solution: *ABCDE* can be divided up into a rectangle and two triangles, each with horizontal bases.

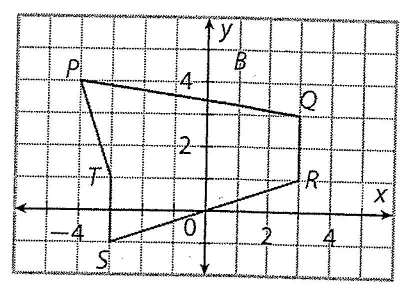
Area = 36 sq. units



(B) *PQRST* can be divided into a parallelogram and a triangle.

$\triangle PQT$ appears to be a right triangle. Check that $\overline{PT}$ and ☐ are perpendicular:

Area = 22 sq. units

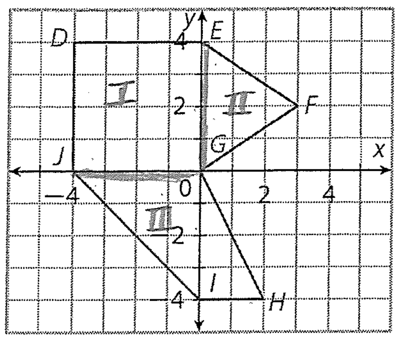


5. **Discussion** How could you use subtraction to find the area of a figure on the coordinate plane?

Draw a rectangle through the outermost vertices;
Find areas of additional figures formed.
Subtract the sum of those areas from the area of the rectangle.

Your Turn

6. Find the area of the polygon by addition.



Area of I:

$A = s^2$

$= 4^2 = 16$

Area of II:

$A = \frac{1}{2}bh$

$= \frac{1}{2}(4)(3)$

$= 6$

Area of III:

$A = \frac{1}{2}h(b_1 + b_2)$

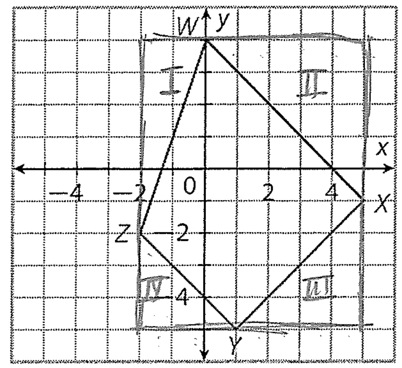
$\frac{1}{2}(4)(4+2)$

$2(6)$

$= 12$

$\boxed{\text{Total area} = 34 \text{ sq. units}}$

7. Find the area of polygon by subtraction.



Area of rectangle $= (7)(9)$

$= 63$

Area of I $= \frac{1}{2}bh$

$= \frac{1}{2}(2)(6)$

$= 6$

Area of II $= \frac{1}{2}(5)(5)$

$= 12\frac{1}{2}$

Area of III $= \frac{1}{2}(4)(4)$

$= 8$

Area of IV $= \frac{1}{2}(3)(3)$

$= 4.5$

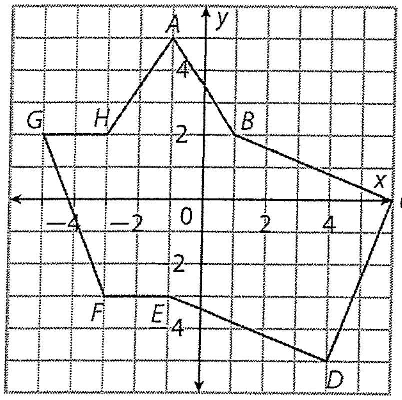
Total area $= 63 - 31$

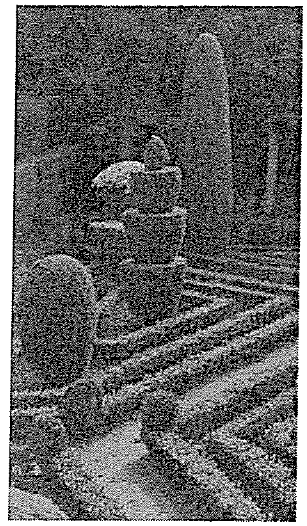
$= \boxed{32 \text{ sq. units}}$

Using Perimeter and Area in Problem Solving

You can use perimeter and area techniques to solve problems.

Example 3 **Miguel is planning and costing an ornamental garden in the a shape of an irregular octagon. Each unit on the coordinate grid represents one yard. He wants to lay the whole garden with turf, which costs \$3.25 per square yard, and surround it with a border of decorative stones, which cost \$7.95 per yard. What is the total cost of the turf and stones?**





Analyze Information

Identify the important information.

- The vertices are A(-1,5) B(1,2) C(6,0) D(4,-5) E(-1,-3) F(-3,-3), G(-5,2), H(-3,2
- The cost of turf is \$ 3.25 per square yard.
- The cost of the ornamental stones is \$ 7.95 per yard.

Formulate a Plan

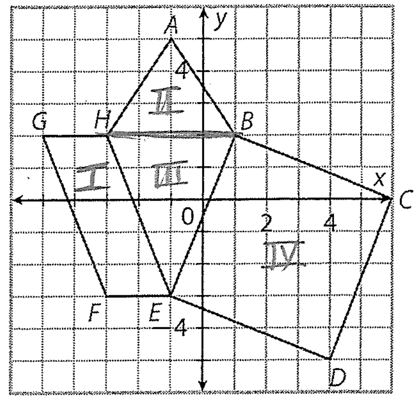
- Divide the garden up into smaller figures
- Add up the areas of the smaller figures.
- Find the cost of turf by multiplying the total area by the cost per square yard.
- Find the perimeter of the garden by adding the lengths of the sides.
- Find the cost of the border by multiplying the perimeter by the cost per yard.
- Find total cost by adding the cost of turf and cost of border

need to verify $AH = AB$ and $HE = BE$

Solve

Divide the garden into smaller figures.

The garden can be divided into square *BCDE*, kite *ABEH*, and parallelogram *EFGH*.



Find the area of each smaller figure.

Verify:

① EFGH is ▱ b/c 1 pr. opp sides are both ∥ & ≅.

② BCDE is a rectangle

$BC = \sqrt{29}$ $DE = \sqrt{29}$

$CD = \sqrt{29}$ $BE = \sqrt{29}$

slope of $\overline{BC} = -\frac{2}{5}$

slope of $\overline{CD} = \frac{5}{2}$

BCDE is a square

~~Area of I~~ $= \overset{bh}{2(5)} = 10$

Area of II $= \frac{1}{2}bh$

$= \frac{1}{2}(4)(3)$

$= 6$

Area of III $= \frac{1}{2}(4)(5)$

$= 10$

Area of IV $= S^2$

$= \sqrt{29}^2$

$= 29$

Total Area = 55 sq yds.

Cost of turf $= 55 \times 3.25 = \$178.75$

$GF = \sqrt{(-3+5)^2 + (-3-2)^2}$

$\sqrt{4+25}$

$\sqrt{29}$

$AH = \sqrt{(-1+3)^2 + (5-2)^2}$

$= \sqrt{13}$

$AB = \sqrt{13}$

Perimeter $= 2\sqrt{13} + 4\sqrt{29} + 2(2)$

≈ 32.75 yds.

Cost of stones $= 32.75 \times 7.95$

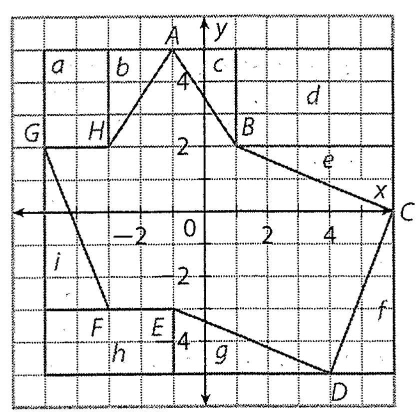
$= \$260.38$

Total cost $= \$439.12$ or $\$439.51$

Justify and Evaluate

The area can be checked by subtraction:

area of large rectangle $= 11(10) = 110$ sq. units

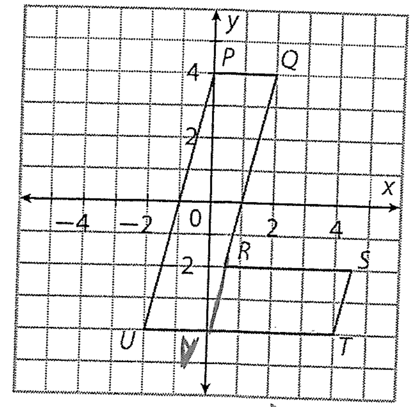


Area $= 110 - (6+3+3+15+5+5+5+8+5)$

$110 - 55$

Area = 55 sq units

8. A designer is making a medallion in the shape of the letter "L." Each unit on the coordinate grid represents an eighth of an inch, and the medallion is to be cut from a 1-in. square of metal. How much metal is wasted to make each medallion? Write your answer as a decimal.



Verify: both figures are $\boxed{\text{▱}}$ b/c 1 pair of sides are both ∥ & ≅.

$\text{Area}_{PQVU} = bh = 2(8) = 16$

$\text{Area}_{RSTV} = 4(2) = 8$

Area of medallion = 24 sq. units

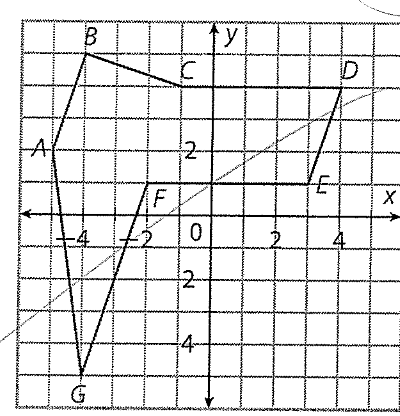
Square of the scale factor:
If 1 unit = $\frac{1}{8}$ inch,
1 sq unit = $\frac{1}{64}$ sq. in.

Area of medallion: $24 \times \frac{1}{64} = \frac{3}{8}$ sq. inch

Area wasted: $1 \text{ sq. in} - \frac{3}{8} = \frac{5}{8}$ sq. in.
≈ 0.625 sq. in.

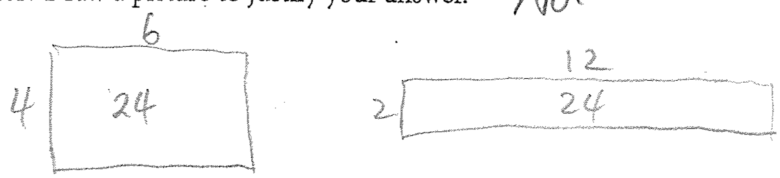
use $R(0.5, -2)$ $S(4\frac{1}{2}, -2)$

9. Create a flowchart for the process of finding the area of the polygon *ABCDEFG*. Your flowchart should show when, and why, the Slope and Distance Formulas are used.



10. Discussion If two polygons have approximately the same area, do they have approximately the same perimeter? Draw a picture to justify your answer. NO.

Per = 20



Perimeter = 24 + 4 = 28

$20 \neq 28$