

Rational Exponent Equations

Solve each equation.

1) $27 = x^2 \left(\frac{2}{3}\right)$

$$3^3 \left(\frac{2}{3}\right) = x$$

$$3^2 = x$$

$$9 = x$$

2) $m^4 = 27 \left(\frac{4}{3}\right)$

$$m^2 = 3^3 \left(\frac{4}{3}\right)$$

$$m^2 = 3^4$$

$$m = 81$$

3) $x^{-\frac{3}{2}} = \frac{1}{729}$

$$x^{-\frac{3}{2}} = 729^{-1}$$

$$x^{-\frac{3}{2} \left(-\frac{2}{3}\right)} = 3^{-6 \left(-\frac{2}{3}\right)}$$

$$x = 3^4 \rightarrow x = 81$$

4) $7 = r^{\frac{1}{2}}$

$$7^2 = r^{\frac{1}{2}(2)}$$

$$49 = r$$

5) $v^{\frac{5}{4}} = 243$

$$\sqrt[5]{v} = 3^5$$

$$\sqrt[5]{v \left(\frac{4}{5}\right)} = 3^{5 \left(\frac{4}{5}\right)}$$

$$v = 3^4 \rightarrow v = 81$$

6) $n^{\frac{3}{2}} = 125$

$$n^{\frac{3}{2}} = 5^3$$

$$n^{\frac{3}{2} \left(\frac{2}{3}\right)} = 5^{3 \left(\frac{2}{3}\right)}$$

$$n = 5^2$$

$$n = 25$$

7) $(n-27)^{\frac{3}{2}} = 64$

$$(n-27)^{\frac{3}{2}} = 4^3$$

$$(n-27)^{\frac{3}{2} \left(\frac{2}{3}\right)} = 4^{3 \left(\frac{2}{3}\right)}$$

$$n-27 = 16$$

$$n = 43$$

8) $26 = -1 + (27x)^4 \frac{3}{4}$

$$27 = (27x)^{\frac{3}{4}}$$

$$3^3 = (27x)^{\frac{3}{4}}$$

$$3^{3 \left(\frac{4}{3}\right)} = (27x)^{\frac{3}{4} \left(\frac{4}{3}\right)}$$

$$3^4 = 27x$$

$$3^4 = 3^3 x$$

$$3 = x$$

$$9) 3125 = (-1 - 18p)^{\frac{5}{3}}$$

$$5^5 = (-1 - 18p)^{\frac{5}{3}}$$

$$5^{5(\frac{3}{5})} = (-1 - 18p)^{\frac{5}{3}(\frac{3}{5})}$$

$$5^3 = -1 - 18p$$

$$126 = -18p \quad \boxed{p = -7}$$

$$11) 4b^{-\frac{3}{4}} + 10 = \frac{21}{2}$$

$$4b^{-\frac{3}{4}} = \frac{1}{2}$$

$$b^{-\frac{3}{4}} = \frac{1}{8}$$

$$b^{-\frac{3}{4}} = 2^{-3}$$

$$b^{-\frac{3}{4}(-\frac{4}{3})} = 2^{-3(-\frac{4}{3})}$$

$$b = 2^4$$

$$\boxed{b = 16}$$

$$13) -54 = 10 - (m - 10)^{\frac{3}{2}}$$

$$(m - 10)^{\frac{3}{2}} = 64$$

$$(m - 10)^{\frac{3}{2}(\frac{2}{3})} = 4^3(\frac{2}{3})$$

$$m - 10 = 16$$

$$\boxed{m = 26}$$

$$-54 = 10 - (16)^{\frac{3}{2}}$$

$$-54 = 10 - 4^2(\frac{3}{2})$$

$$-54 = 10 - 4^3$$

$$-54 = 10 - 64$$

$$15) 9 + 5\sqrt[3]{2m} = 29$$

$$5\sqrt[3]{2m} = 20$$

$$(\sqrt[3]{2m})^3 = 4^3$$

$$2m = 64$$

$$\boxed{m = 32}$$

$$17) -646 = -3(65 - n)^{\frac{3}{2}} + 2$$

$$-648 = -3(65 - n)^{\frac{3}{2}}$$

$$216 = (65 - n)^{\frac{3}{2}}$$

$$6^2 = (65 - n)^{\frac{3}{2}}$$

$$6^{3(\frac{2}{3})} = (65 - n)^{\frac{3}{2}(\frac{2}{3})}$$

$$6^2 = 65 - n$$

$$\boxed{n = 29}$$

$$10) 5 = 3 + 4a^{-\frac{1}{6}}$$

$$2 = 4a^{-\frac{1}{6}}$$

$$\left(\frac{1}{2}\right)^{-6} = a^{-\frac{1}{6}(-6)}$$

$$2^6 = a$$

$$\boxed{a = 64}$$

$$5 \stackrel{?}{=} 3 + 4(2^6)^{\frac{1}{6}}$$

$$= 3 + 4(2^{-1})$$

$$= 3 + 4\left(\frac{1}{2}\right)$$

$$5 = 5$$

$$12) -x^{\frac{3}{2}} = -27$$

$$x^{\frac{3}{2}} = 3^3$$

$$x^{\frac{3}{2}(\frac{2}{3})} = 3^{3(\frac{2}{3})}$$

$$\boxed{x = 9}$$

$$14) -5126 = -6 - 5(3x + 22)^{\frac{5}{3}}$$

$$-5120 = -5(3x + 22)^{\frac{5}{3}}$$

$$1024 = (3x + 22)^{\frac{5}{3}}$$

$$4^{5(\frac{3}{5})} = (3x + 22)^{\frac{5}{3}(\frac{3}{5})}$$

$$4^3 = 3x + 22$$

$$42 = 3x \rightarrow \boxed{x = 14}$$

$$16) 3646 = 1 + 5(4r + 17)^{\frac{3}{2}}$$

$$3645 = 5(4r + 17)^{\frac{3}{2}}$$

$$729 = (4r + 17)^{\frac{3}{2}}$$

$$9^{3(\frac{2}{3})} = (4r + 17)^{\frac{3}{2}(\frac{2}{3})}$$

$$81 = 4r + 17$$

$$\boxed{r = 16}$$

$$18) -3 + (8 - 2x)^4 = 29$$

$$(8 - 2x)^{\frac{5}{4}} = 32$$

$$(8 - 2x)^{\frac{5}{4}(\frac{4}{5})} = 2^5(\frac{4}{5})$$

$$8 - 2x = 2^4$$

$$8 - 2x = 16$$

$$-2x = 8$$

$$\boxed{x = -4}$$