

Solving Radical Inequalities

Solve each inequality.

1) $3\sqrt{x-4} \geq 5$

$3\sqrt{x} \geq 9$

$\sqrt{x} \geq 3$

$x \geq 9$

$\{x \geq 9\}$

$$\begin{array}{l} \text{CK} \\ x=9 \quad x=16 \\ 3\sqrt{9}-4 \geq 5 \quad 3\sqrt{16}-4 \\ 3(3)-4 \quad 3(4)-4 \\ 5 \geq 5 \quad 8 \geq 5 \\ \checkmark \quad \checkmark \end{array}$$

2) $\sqrt{x-3} \leq 7$

$x-3 \leq 49$

$x \leq 52$

$x-3 \geq 0$

$x \geq 3$

$\{3 \leq x \leq 52\}$

3) $5\sqrt{x-1} > 10$

$\sqrt{x-1} > 2$

$x-1 > 4$

$x > 5$

$\{x > 5\}$

$x-1 > 0$

$x > 1$

4) $4\sqrt{x+3} \leq 23$

$4\sqrt{x} \leq 20$

$\sqrt{x} \leq 5$

$x \leq 25$

$\{0 \leq x \leq 25\}$

CK

$4\sqrt{25} + 3 \stackrel{?}{\leq} 23$

$4(5) + 3$

$23 \leq 23$

5) $\sqrt{x+10} \geq 6$

$x+10 \geq 36$

$x \geq 26$

$\{x \geq 26\}$

6) $-3\sqrt{x+2} < 15$

$\sqrt{x+2} > -5$

$x+2 > 25$

$x > 23$

$\{-2 \leq x\}$

$x+2 \geq 0$

$x \geq -2$

CK

If $x = -3$

$-3\sqrt{0} < 15$

If $x = 23$

$-3\sqrt{25} < 15$

$-3(5) < 15$

7) $\sqrt{x-3} \geq 4$

$x-3 \geq 16$

$x \geq 19$

$\{x \geq 19\}$

8) $(\sqrt{x+4})^2 \geq (2\sqrt{x})^2$

$x+4 \geq 4x$

$4 \geq 3x$

$\frac{4}{3} \geq x$

$\{0 \leq x \leq \frac{4}{3}\}$

CK

$\sqrt{\frac{4}{3}+4} \stackrel{?}{\geq} 2\sqrt{\frac{4}{3}}$

$\sqrt{\frac{16}{3}} \geq 2\sqrt{\frac{4}{3}}$

$\frac{4}{\sqrt{3}} \geq \frac{4}{\sqrt{3}} \checkmark$

$x=0$

$\sqrt{4} \geq 2\sqrt{0}$

$2 \geq 0 \checkmark$

Solve each radical inequality by using algebra. If the inequality has no real solution, write *no solution*. Check your solution.

25. $\sqrt{2x-1} \geq 1$

27. $\sqrt{2x-1} \leq 1$

29. $-\sqrt{x} \leq 2$

31. $1 > 3\sqrt{3x-1}$

33. $3\sqrt{3x-1} < -1$

35. $\sqrt{2x+2} > \sqrt{3x}$

37. $x \leq \sqrt{x}$

26. $\sqrt{3x+4} \geq 2$

28. $\sqrt{3x+4} \leq 2$

30. $-\sqrt{x} \geq 2$

32. $\sqrt{2x+1} - 3 < 0$

34. $\sqrt{2x+1} + 3 < 0$

36. $\sqrt{9x+7} < \sqrt{14x}$

38. $\frac{1}{8} \leq x \leq \sqrt{x}$

(25) $2x-1 \geq 1$ $2x-1 \geq 0$
 $2x \geq 2$ $2x \geq 1$
 $x \geq 1$ $x \geq \frac{1}{2}$
 $\{x \geq 1\}$

(26) $3x+4 \geq 4$
 $3x \geq 0$
 $x \geq 0$
 $\{x \geq 0\}$

(27) $2x-1 \leq 1$ $2x-1 \geq 0$
 $2x \leq 2$ $2x \geq 1$
 $x \leq 1$ $x \geq \frac{1}{2}$
 $\{\frac{1}{2} \leq x \leq 1\}$

(28) $3x+4 \leq 4$ $3x+4 \geq 0$
 $3x \leq 0$ $3x \geq -4$
 $x \leq 0$ $x \geq -\frac{4}{3}$
 $\{-\frac{4}{3} \leq x \leq 0\}$

(29) $-\sqrt{x} \leq 2$
 $\{x \geq 0\}$

(30) $-\sqrt{x} \geq 2$ (31) $\frac{1}{3} > \sqrt{3x-1}$ $3x-1 \geq 0$
 $\{ \}$ $\frac{1}{9} > 3x-1$ $x \geq \frac{1}{3}$
 $\frac{10}{9} > 3x$
 $\frac{10}{27} > x$
 $\{\frac{1}{3} \leq x < \frac{10}{27}\}$

(32) $\sqrt{2x+1} < 3$ $2x+1 > 0$
 $2x+1 < 9$ $2x > -1$
 $2x < 8$ $x > -\frac{1}{2}$
 $x < 4$
 $\{-\frac{1}{2} < x < 4\}$

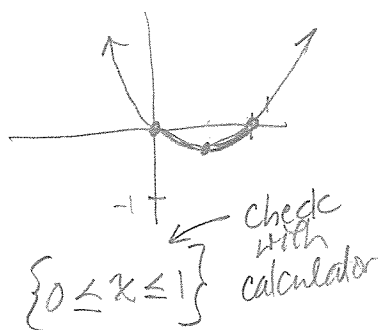
(33) $\sqrt{3x-1} < -\frac{1}{3}$ $3x-1 \geq 0$
 $3x-1 < \frac{1}{9}$ $x \geq \frac{1}{3}$
 $3x < \frac{10}{9}$
 $x < \frac{10}{27}$
 $\{\frac{1}{3} \leq x < \frac{10}{27}\}$

(34) $\sqrt{2x+1} < -3$ $2x+1 \geq 0$
 $2x+1 < 9$ $x \geq -\frac{1}{2}$
 $2x < 8$
 $x < 4$
 $\{-\frac{1}{2} \leq x < 4\}$

(35) $2x+2 > 3x$ (36) $9x+7 < 14x$
 $2 > x$ $7 < 5x$
 $2x+2 \geq 0$ $\frac{7}{5} < x$
 $x \geq -1$ $\{x > \frac{7}{5}\}$
 but $3x \geq 0$
 $x \geq 0$
 $\{0 \leq x < 2\}$

ck
 $x = \frac{7}{5}$
 $\sqrt{9 \cdot \frac{7}{5} + 7} < \sqrt{14 \cdot \frac{7}{5}}$
 $\sqrt{\frac{98}{5}} \neq \sqrt{\frac{98}{5}}$
 $x = 2$
 $\sqrt{9 \cdot 2 + 7} < \sqrt{14 \cdot 2}$
 $\sqrt{25} < \sqrt{28}$

(37) $x \leq \sqrt{x}$
 $x^2 \leq x$
 $x^2 - x \leq 0$
 $x(x-1) \leq 0$
 $-\frac{1}{2}$
 $\frac{1}{4} - \frac{1}{2}$



(38) $x \leq \sqrt{x}$ is same as #37:
 $0 \leq x \leq 1$ and
 $\frac{1}{8} \leq x$
 \Downarrow
 $\frac{1}{8} \leq x \leq 1$

ck
 $\frac{1}{8} \leq \frac{1}{8} \leq \sqrt{\frac{1}{8}}$
 $.125 \leq .125 \leq .3535 \checkmark$
 $.125 \leq .75 \leq .866$