

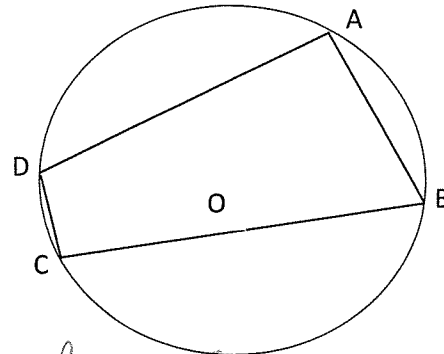
Proving the Inscribed Quadrilateral Theorem

Inscribed Quadrilateral Theorem:

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Given: ABCD is inscribed in circle O.

Prove: $\angle A$ and $\angle C$ are supplementary.
 $\angle B$ and $\angle D$ are supplementary.



Statements

- ① Union of \widehat{BCD} and \widehat{DAB} is circle O.
- ② $m\widehat{BCD} + m\widehat{DAB} = 360^\circ$
- ③ $\angle A$ is an inscribed \angle and its intercepted arc is \widehat{BCD} .
 $\angle C$ is an inscribed \angle and its intercepted arc is \widehat{DAB} .
- ④ $m\angle A = \frac{1}{2} m\widehat{BCD}$
 $m\angle C = \frac{1}{2} m\widehat{DAB}$
- ⑤ $m\angle A + m\angle C = \frac{1}{2} m\widehat{BCD} + \frac{1}{2} m\widehat{DAB}$
- ⑥ $= \frac{1}{2} (m\widehat{BCD} + m\widehat{DAB})$
- ⑦ $= \frac{1}{2} (360)$
- ⑧ $m\angle A + m\angle C = 180^\circ$

Reasons

- ① A whole is equal to sum of its parts. or Arc Addition Postulate
 - ② Substitution.
 - ③ Definition of inscribed \angle and intercepted arc.
 - ④ An inscribed \angle is $\frac{1}{2}$ of its intercepted arc.
 - ⑤ Add. Property
 - ⑥ Distributive Prop.
 - ⑦ Substitution Prop.
- ← Repeat Steps #1-8 using \widehat{ABC} & \widehat{ADC}

The converse of the Inscribed Quadrilateral Theorem is also true.

Theorem: If the opposite angles of a quadrilateral are supplementary, then the quadrilateral can be inscribed in a circle.