Chapter 6 6.1

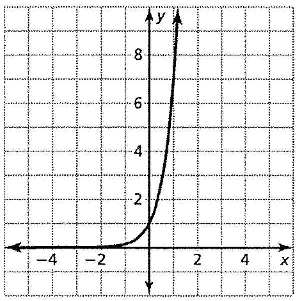
10. Step 1 Identify the value of the base. The base, 7, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	-2	-1	0	1
y	$\frac{1}{49}$	$\frac{1}{7}$	1	7

Step 3 Plot the points from the table.

Step 4 Draw, from left to right, a smooth curve that begins above the x-axis, passes through the plotted points, and moves up to the right.



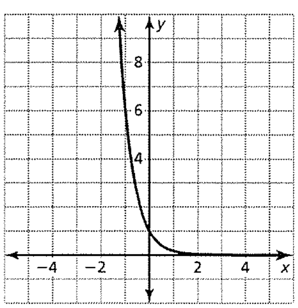
11. Step 1 Identify the value of the base. The base, $\frac{1}{6}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-1	0	1	2
y	6	1	$\frac{1}{6}$	$\frac{1}{36}$

Step 3 Plot the points from the table.

Step 4 Draw, from right to left, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



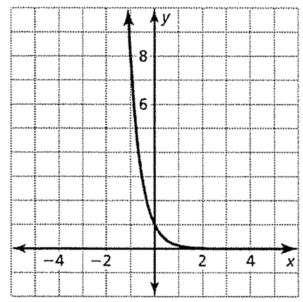
12. Step 1 Identify the value of the base. The base, $\frac{1}{8}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-1	0	1	2
y	8	1	$\frac{1}{8}$	$\frac{1}{64}$

Step 3 Plot the points from the table.

Step 4 Draw, from right to left, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



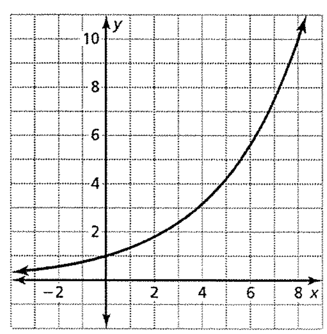
13. Step 1 Identify the value of the base. The base, $\frac{4}{3}$, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	-2	-1	0	1	2	3
y	$\frac{9}{16}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\frac{16}{9}$	$\frac{64}{27}$

Step 3 Plot the points from the table.

Step 4 Draw, from left to right, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.



Chapter 6

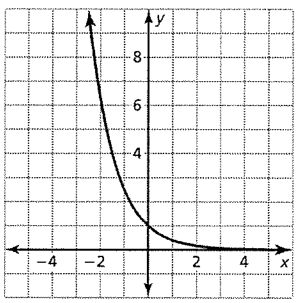
14. Step 1 Identify the value of the base. The base, $\frac{2}{5}$, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-2	-1	0	1	2
y	$\frac{25}{4}$	$\frac{5}{2}$	1	$\frac{2}{5}$	$\frac{4}{25}$

Step 3 Plot the points from the table.

Step 4 Draw, from right to left, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



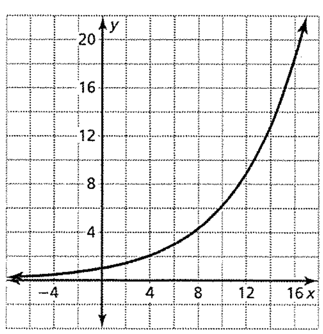
15. Step 1 Identify the value of the base. The base, 1.2, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	-2	-1	0	1	2	3
y	$0.69\overline{4}$	$0.8\overline{3}$	1	1.2	1.44	1.728

Step 3 Plot the points from the table.

Step 4 Draw, from left to right, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the right.



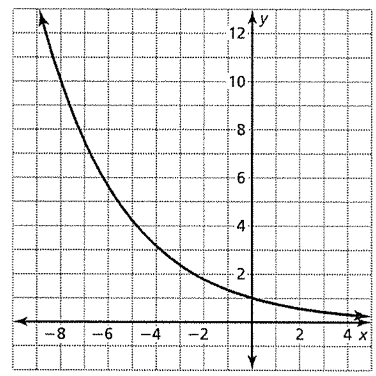
16. Step 1 Identify the value of the base. The base, 0.75, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-3	-2	-1	0	1	2
y	$2.\overline{370}$	$1.\overline{7}$	$1.\overline{3}$	1	0.75	0.5625

Step 3 Plot the points from the table.

Step 4 Draw, from right to left, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



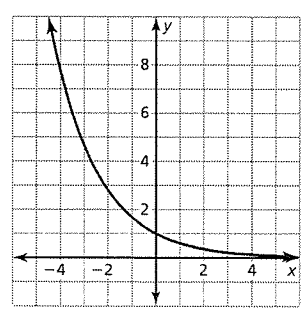
17. Step 1 Identify the value of the base. The base, 0.6, is greater than 0 and less than 1, so the function represents exponential decay.

Step 2 Make a table of values.

x	-2	-1	0	1	2	3
y	$2.\overline{7}$	$1.\overline{6}$	1	0.6	0.36	0.216

Step 3 Plot the points from the table.

Step 4 Draw, from right to left, a smooth curve that begins just above the x-axis, passes through the plotted points, and moves up to the left.



Chapter 6

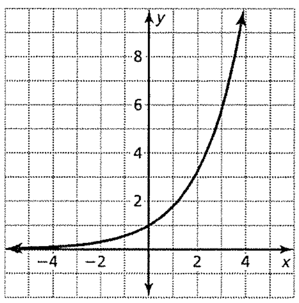
18. Step 1 Identify the value of the base. The base, 1.8, is greater than 1, so the function represents exponential growth.

Step 2 Make a table of values.

x	-1	0	1	2	3
y	$0.\overline{5}$	1	1.8	3.24	5.832

Step 3 Plot the points from the table.

Step 4 Draw, from left to right, a smooth curve that begins just above the x-axis, passes through the plotted points, and move up to the right.



19. The base b is 3 because the value of the function is 3 when $x = 1$.

20. The base b is 5 because the value of the function is 5 when $x = 1$.

21. a. The base, 0.75, is greater than 0 and less than 1, so the model represents exponential decay.

b. Because t is given in years and the decay factor is $0.75 = 1 - 0.25$, the annual percent decrease is 0.25, or 25%.

c. Use the *trace* feature of a graphing calculator to determine that $y \approx 50$ when $t = 4.8$. After 4.8 years, the value of the bike will be about \$50.

22. a. The base, 1.03, is greater than 1, so the model represents exponential growth.

b. Because t is given in years and the growth factor is $1.03 = 1 + 0.03$, the annual percent increase is 0.03, or 3%.

c. Use the *trace* feature of a graphing calculator to determine that $y \approx 590$ when $t = 6$. The population was about 590,000 six years after the beginning of the decade.

23. a. The initial amount is $a = 233$, and the percent increase is $r = 0.06$. So, the exponential growth model is

$$y = a(1 + r)^t$$
$$= 233(1 + 0.06)^t$$
$$= 233(1.06)^t.$$

Using this model, you can determine the number of cell phone subscribers in 2008, when $t = 2$, to be $y = 233(1.06)^2 \approx 261.8$ million.

b. Use the *table* feature of a graphing calculator to determine that $y \approx 278$ when $t = 3$. So, there were about 278 million cell phone subscribers in 2009.

24. a. The initial amount is $a = 325$, and the percent decrease is $r = 0.29$. So, the exponential decay model is

$$y = a(1 - r)^t$$
$$= 325(1 - 0.29)^t$$
$$= 325(0.71)^t$$

b. Use the *table* feature of a graphing calculator to determine that $y \approx 100$ when $t = 3.4$. So, after about 3.4 hours there is about 100 milligrams of ibuprofen in your blood stream.

25.

$y = a(3)^{t/14}$	Write original function.
$= a[(3)^{1/14}]^t$	Power of a Power Property.
$\approx a(1.0816)^t$	Evaluate power.
$= a(1 + 0.0816)^t$	Rewrite in form $y = a(1 + r)^t$.

26.

$y = a(0.1)^{t/3}$	Write original function.
$= a[(0.1)^{1/3}]^t$	Power of a Power Property.
$\approx a(0.4642)^t$	Evaluate power.
$= a(1 - 0.5358)^t$	Rewrite in form $y = a(1 - r)^t$.

27. $y = a(0.5)^{t/5730}$

$$= a[(0.5)^{1/5730}]^t$$
$$\approx a(0.9999)^t$$
$$= a(1 - 0.0001)^t$$

The yearly decay rate is about 0.0001, or 0.01%.

28. $y = a(1230.25)^{t/16}$

$$= a[(1230.25)^{1/16}]^t$$
$$\approx a(1.56)^t$$
$$= a(1 + 0.56)^t$$

The daily growth rate is about 0.56, or 56%.

29. $y = a(2)^{t/3}$

$$= a[(2)^{1/3}]^t$$
$$\approx a(1.26)^t$$
$$= a(1 + 0.26)^t$$

The growth rate is about 0.26, or 26%.

Chapter 6

30. $y = a(4)^{t/6}$
$= a[(4)^{1/6}]^t$
$\approx a(1.26)^t$
$= a(1 + 0.26)^t$

The growth rate is about 0.26, or 26%.

31. $y = a(0.5)^{t/12}$
$= a[(0.5)^{1/12}]^t$
$\approx a(0.94)^t$
$= a(1 - 0.06)^t$

The decay rate is about 0.06, or 6%.

32. $y = a(0.25)^{t/9}$
$= a[(0.25)^{1/9}]^t$
$\approx a(0.86)^t$
$= a(1 - 0.14)^t$

The decay rate is about 0.14, or 14%

33. $y = a\left(\frac{2}{3}\right)^{t/10}$
$= a\left[\left(\frac{2}{3}\right)^{1/10}\right]^t$
$\approx a(0.96)^t$
$= a(1 - 0.04)^t$

The decay rate is about 0.04, or 4%

34. $y = a\left(\frac{5}{4}\right)^{t/22}$
$= a\left[\left(\frac{5}{4}\right)^{1/22}\right]^t$
$\approx a(1.01)^t$
$= a(1 + 0.01)^t$

The growth rate is about 0.01, or 1%.

35. $y = a(2)^{8t}$
$= a[(2)^8]^t$
$= a(256)^t$
$= a(1 + 255)^t$

The growth rate is 255, or 25,500%.

36. $y = a\left(\frac{1}{3}\right)^{3t}$
$= a\left[\left(\frac{1}{3}\right)^3\right]^t$
$\approx a(0.04)^t$
$= a(1 - 0.96)^t$

The decay rate is about 0.96, or 96%.

37. With interest compounded quarterly (4 times per year), the balance after 5 years is

$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$= 5000\left(1 + \frac{0.0225}{4}\right)^{4 \cdot 5}$
$\approx 5593.60.$

The balance at the end of 5 years is \$5593.60.

38.

Account	**Compounding**	**Balance after 6 years**
1	quarterly	$A = 2200\left(1 + \frac{0.03}{4}\right)^{4 \cdot 6} \approx 2632.11$
2	monthly	$A = 2200\left(1 + \frac{0.03}{12}\right)^{12 \cdot 6} \approx 2633.29$
3	daily	$A = 2200\left(1 + \frac{0.03}{365}\right)^{365 \cdot 6} \approx 2633.86$

Account 1 earns about \$2632.11 − \$2200 = \$432.11 in interest.

Account 2 earns about \$2633.29 − \$2200 = \$433.29 in interest.

Account 3 earns about \$2633.86 − \$2200 = \$433.86 in interest.

39. The percent decrease was used as the decay factor. The decay factor is 1 − 0.02 = 0.98.

$y = (\text{initial amount})(\text{decay factor})^t$

$y = 500(0.98)^t$

40. The percent was not changed to its decimal form of 0.0125.

$A = 250\left(1 + \frac{0.0125}{4}\right)^{4 \cdot 3}$

$A = \$259.54$

41. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$= 3500\left(1 + \frac{0.0216}{4}\right)^{4 \cdot 6}$
$= \$3982.92$

42. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$= 3500\left(1 + \frac{0.0229}{12}\right)^{12 \cdot 6}$
$= \$4014.98$

43. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$= 3500\left(1 + \frac{0.0183}{365}\right)^{365 \cdot 6}$
$= \$3906.18$

Chapter 6

44. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 3500\left(1 + \frac{0.0126}{12}\right)^{12 \cdot 6}$

$= \$3774.71$

45. The value of a represents the initial number of referrals. So, the website received 2500 referrals at the start of the 10-year period. The value of b represents the growth factor. The growth factor is $1.50 = 1 + 0.50$, so the annual percent increase is 0.50, or 50%.

46. a. The graph represents exponential decay.

b. Because $f(x)$ is an exponential function and any real number can be used as an exponent, the domain is all real numbers. Because $f(x) \to 0$ as $x \to \infty$ and $f(x) \to \infty$ as $x \to -\infty$, the range is $y > 0$.

47. Your friend is incorrect. $f(x) = 2^x$ does not have a faster growth rate for $x \geq 0$. For instance, the secant line between (1, 1) and (2, 4) on $g(x) = x^2$ is steeper than the secant line between (1, 2) and (2, 4) on $f(x)$.

48. *Sample answer:* The function $g(x) = (1 - b)^x$ is an exponential decay function because $1 - b < 1$.

49. The average rate of change over the first 6 years is $\frac{6850(1.03)^6 - 6850(1.03)^0}{6 - 0} \approx \frac{8179 - 6850}{6} = 221.5.$ So, the average rate of change over the first 6 years is about 221.5 people per year.

50. a. $\frac{f(x+1)}{f(x)} = \frac{ab^{x+1}}{ab^x}$

$= \frac{b^{x+1}}{b^x}$

$= b^{x+1-x}$

$= b$

b. By part (a), the ratios of consecutive terms must be equal for the points to be modeled by an exponential function.

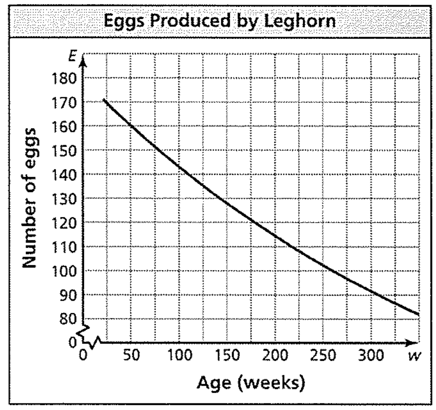
$\frac{f(1)}{f(0)} = \frac{4}{4} = 1$

and

$\frac{f(2)}{f(1)} = \frac{8}{4} = 2$

The ratios are not equal, so there is no exponential function of the form $f(x) = ab^x$ whose graph passes through the points in the table.

51. a. The decay factor is 0.89 and the percent decrease is 0.11, or 11%.

b. 

c. A chicken that is 2.5 years old is $2.5(52) = 130$ weeks old and produces about $E = 179.2(0.89)^{130/52} \approx 134$ eggs per year.

d. To use years, rather than weeks, replace $\frac{w}{52}$ with y, where y is the age in years.

52. Use the model $V = ab^t$. When the stereo is new, $t = 0$ and $V = 1300$, so $1300 = ab^0 = a$. After 4 years, $V = 275$, so $275 = 1300b^4$. Solve for b.

$275 = 1300b^4$

$0.2115 = b^4$

$0.6782 \approx b$

So, the model for the value of stereo is $V = 1300(0.6782)^t$.

Maintaining Mathematical Proficiency

53. $x^9 \cdot x^2 = x^{9+2} = x^{11}$

54. $\frac{x^4}{x^3} = x^{4-3} = x$

55. $4x \cdot 6x = 24x^{1+1} = 24x^2$

56. $\left(\frac{4x^8}{2x^6}\right)^4 = (2x^{8-6})^4 = (2x^2)^4 = 16x^8$

57. $\frac{x + 3x}{2} = \frac{4x}{2} = 2x$

58. $\frac{6x}{2} + 4x = 3x + 4x = 7x$

59. $\frac{12x}{4x} + 5x = 3 + 5x$

60. $(2x \cdot 3x^5)^3 = (6x^6)^3 = 216x^{18}$

6.2 Explorations (p. 303)

1. $e \approx 2.718$; *Sample answer:* Add a few more values to the sequence shown by following its pattern. Use a graphing calculator to find the value of each fraction, and add these values; 3