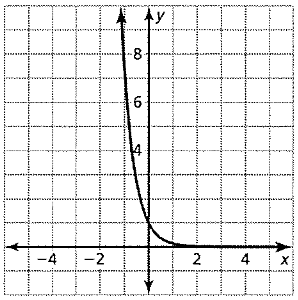
Chapter 6

16. Because $a = 1$ is positive and $r = -2$ is negative, the function is an exponential decay function.

Use a table to graph the function.

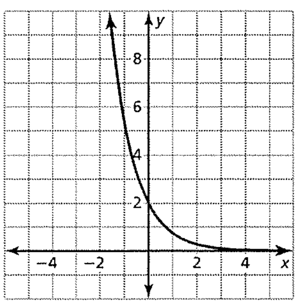
x	-2	-1	0	1
y	54.6	7.39	1	0.14



17. Because $a = 2$ is positive and $r = -1$ is negative, the function is an exponential decay function.

Use a table to graph the function.

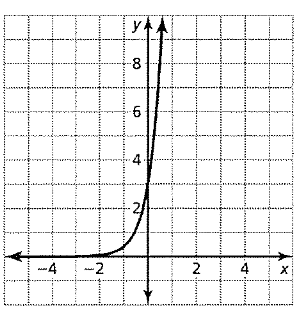
x	-2	-1	0	1
y	14.78	5.44	2	0.74



18. Because $a = 3$ is positive and $r = 2$ is positive, the function is an exponential growth function.

Use a table to graph the function.

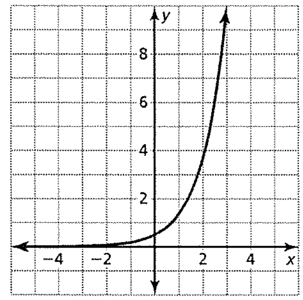
x	-2	-1	0	1
y	0.05	0.41	3	22.17



19. Because $a = 0.5$ is positive and $r = 1$ is positive, the function is an exponential growth function.

Use a table to graph the function.

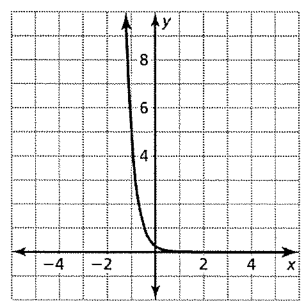
x	-2	-1	0	1	2
y	0.07	0.18	0.5	1.36	3.69



20. Because $a = 0.25$ is positive and $r = -3$ is negative, the function is an exponential decay function.

Use a table to graph the function.

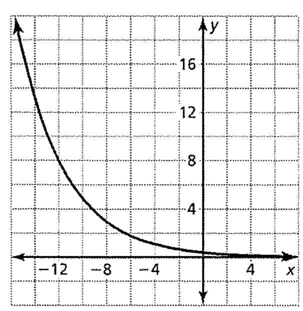
x	-2	-1	0	1
y	100.86	5.02	0.25	0.01



21. Because $a = 0.4$ is positive and $r = -0.25$ is negative, the function is an exponential decay function.

Use a table to graph the function.

x	-2	-1	0	1
y	0.66	0.51	0.4	0.31

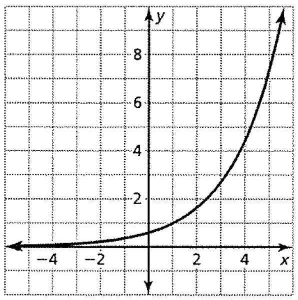


Chapter 6

22. Because $a = 0.6$ and $r = 0.5$ are positive, the function is an exponential growth function.

Use a table to graph the function.

x	-2	-1	0	1
y	0.22	0.36	0.6	0.99



23. The graph is D because the function represents exponential growth and (0, 1) is a point on the graph of the function.

24. The graph is A because the function represents exponential decay and (0, 1) is a point on the graph of the function.

25. The graph is B because the function represents exponential decay and (0, 4) is a point on the graph of the function.

26. The graph is C because the function represents exponential growth and (0, 0.75) is a point on the graph of the function.

27. $y = e^{-0.25t}$

$= (e^{-0.25})^t$

$\approx (0.779)^t$

$= (1 - 0.221)^t$

The percent decrease is about 22.1%.

28. $y = e^{-0.75t}$

$= (e^{-0.75})^t$

$\approx (0.472)^t$

$= (1 - 0.528)^t$

The percent decrease is about 52.8%.

29. $y = 2e^{0.4t}$

$= 2(e^{0.4})^t$

$\approx 2(1.492)^t$

$= 2(1 + 0.492)^t$

The percent increase is about 49.2%.

30. $y = 0.5e^{0.8t}$

$= 0.5(e^{0.8})^t$

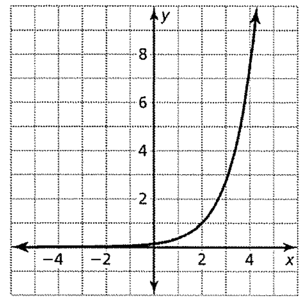
$\approx 0.5(2.226)^t$

$= 0.5(1 + 1.226)^t$

The percent increase is about 122.6%.

31.

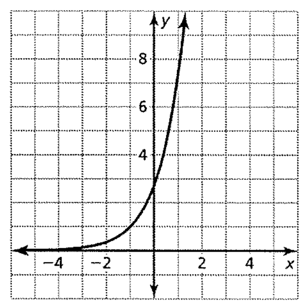
x	-2	-1	0	1	2
y	0.02	0.05	0.14	0.37	1



The domain of the function is all real numbers and the range of the function is $y > 0$.

32.

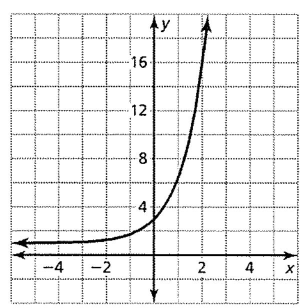
x	-2	-1	0	1	2
y	0.37	1	2.72	7.39	20.09



The domain of the function is all real numbers and the range of the function is $y > 0$.

33.

x	-2	-1	0	1
y	1.27	1.74	3	6.44



The domain of the function is all real numbers and the range of the function is $y > 1$.

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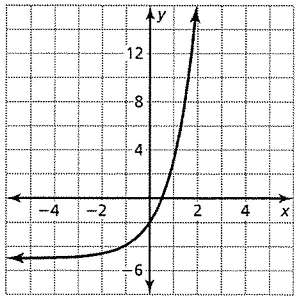
34.

x	-2	-1	0	1
y	-4.59	-3.90	-2	3.15



The domain of the function is all real numbers and the range of the function is $y > -5$.

35. **Understand the Problem** You are given a graph and an equation that represent account balances. You are asked to identify the account with the greater principal and the account with the greater balance after 10 years.

Make a Plan Use the equation to find the principal and account balance of the house fund after 10 years. Then compare these values to the graph of the education account.

Solve the Problem The equation $H = 3224e^{0.05t}$ is of the form $H = Pe^{rt}$, where $P = 3224$. So, the principal of the house account is \$3224. The balance when $t = 10$ is $H = 3224e^{0.05(10)} \approx \$5315.48$.

Because the graph passes through (0, 4856), the principal of the education account is \$4856. The graph also shows that the balance is about \$7250 when $t = 10$.

So, the education account has the greater principal and greater balance after 10 years than the house account.

36. **Understand the Problem** You are given a graph and an equation that represent amounts of elements in a sample over time. You are asked to identify the sample that started with a greater amount and the sample that has a greater amount after 10 years.

Make a Plan Use the equation to calculate the starting amount and remaining amount of tritium after 10 years. Then compare these values to the graph of sodium-22 decay.

Solve the Problem The equation $y = 10e^{-0.0562t}$ is of the form $y = Pe^{rt}$, where $P = 10$. So, the starting amount of the tritium sample is 10 milligrams.

The amount remaining after 10 years is

$y = 10e^{-0.0562(10)} \approx 5.7$ milligrams.

Because the graph passes through (0, 15), the starting amount of sodium-22 in a sample is 15 milligrams. The graph also shows that the remaining amount after 10 years is about 1.25 milligrams.

So, sodium-22 started with a greater amount than tritium, but the tritium sample has a greater amount after 10 years than the sodium-22 sample.

37. *Sample answer:* $a = 1, b = 2, r = -2, q = -5$. So, $f(x) = e^{-2x}$ and $g(x) = 2e^{-5x}$ are exponential decay functions, and $\frac{f(x)}{g(x)} = \frac{e^{-2x}}{2e^{-5x}} = \frac{1}{2}e^{3x}$ is an exponential growth function.

38. Let $m = \frac{n}{r}$, so $n = mr$ and $\frac{r}{n} = \frac{1}{m}$.

Substituting into $A = P\left(1 + \frac{r}{n}\right)^{nt}$ gives $A = P\left(1 + \frac{1}{m}\right)^{mrt}$

which can be written as $A = P\left[\left(1 + \frac{1}{m}\right)^{m}\right]^{rt}$. By definition, $\left(1 + \frac{1}{m}\right)^{m}$ approaches e as m approaches $+\infty$. So, the equation becomes $A = Pe^{rt}$.

39. no; The natural base e is an irrational number, so it cannot be written as the ratio of two integers.

40. Your friend is incorrect. Because $y = f(x)$ is an exponential function, it does not have an x-intercept.

41. For account 1, $P = 2500$, $n = 4$, $t = 10$, and $r = 0.06$, so the balance in 10 years is

$A = 2500\left(1 + \frac{0.06}{4}\right)^{40}$

$\approx \$4535.05$.

For account 2, $P = 2500$, $t = 10$, and $r = 0.04$, so the balance in 10 years is

$A = 2500e^{0.04(10)}$

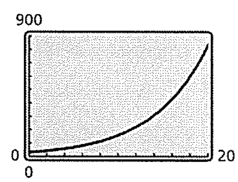
$\approx \$3729.56$.

So, you should choose account 1 to obtain the greater amount in 10 years.

42. a. $f(x)$ increases as x increases, so $f(x)$ approaches $+\infty$ as x approaches $+\infty$.

b. $f(x)$ approaches -3 as x approaches $-\infty$.

43. a. After 1:00 P.M. the number of bacteria is given by the function $N(t) = 30e^{0.166t}$.

b. 

c. At 3:45 P.M., $t = 2.75$ hours.

So, to find the number of cells in the sample at 3:45 P.M., substitute 2.75 for t in the equation in part (a) and simplify.

At 3:45 P.M., there are about 47 cells.

Maintaining Mathematical Proficiency

44. $0.006 = 6 \times 10^{-3}$

45. $5000 = 5 \times 10^{3}$

46. $26{,}000{,}000 = 2.6 \times 10^{7}$

47. $0.000000047 = 4.7 \times 10^{-8}$

Chapter 6

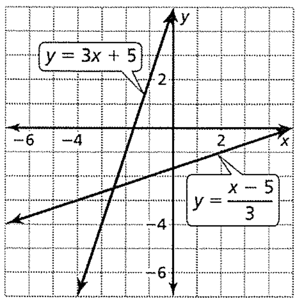
48. $y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$\frac{x - 5}{3} = y$

So, the inverse function is $y = \frac{x - 5}{3}$.



49. $y = x^2 - 1, x \le 0$

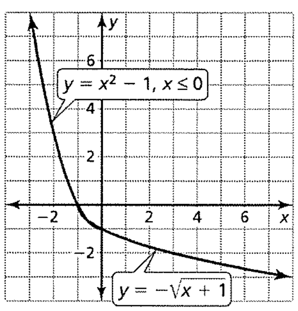
$x = y^2 - 1, y \le 0$

$x + 1 = y^2$

$\pm\sqrt{x + 1} = y$

Because $x \le 0$ in the original equation, the range of the inverse is $y \le 0$. So, choose the negative root.

So, the inverse function is $y = -\sqrt{x + 1}$.



50. $y = \sqrt{x + 6}$

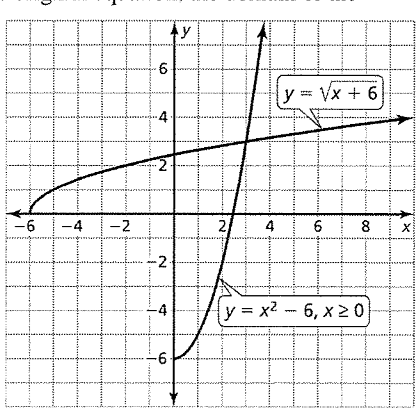
$x = \sqrt{y + 6}$

$x^2 = y + 6$

$x^2 - 6 = y$

Because $y \ge 0$ in the original equation, the domain of the inverse is $x \ge 0$.

So, the inverse function is $y = x^2 - 6, x \ge 0$.



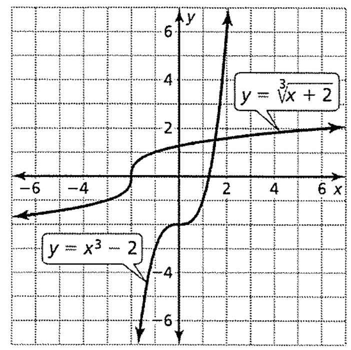
51. $y = x^3 - 2$

$x = y^3 - 2$

$x + 2 = y^3$

$\sqrt[3]{x + 2} = y$

So, the inverse function is $y = \sqrt[3]{x + 2}$.



6.3 Explorations (p. 309)

1. a. The value of x is 3 because $2^3 = 8$; $3 = \log_2 8$.

b. The value of x is 2 because $3^2 = 9$; $2 = \log_3 9$.

c. The value of x is $\frac{1}{2}$ because $4^{1/2} = 2$; $\frac{1}{2} = \log_4 2$.

d. The value of x is 0 because $5^0 = 1$; $0 = \log_5 1$.

e. The value of x is -1 because $5^{-1} = \frac{1}{5}$; $-1 = \log_5 \frac{1}{5}$.

f. The value of x is $\frac{2}{3}$ because $8^{2/3} = 4$; $\frac{2}{3} = \log_8 4$.

2. a.

x	-2	-1	0	1	2
$f(x) = 2^x$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$

The functions f and g are inverses, so the x- and y-coordinates can be switched to create the table for g.

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$g(x) = \log_2 x$	-2	-1	0	1	2