Chapter 6

6.4 Exercises (pp. 322–324)

Vocabulary and Core Concept Check

1. Positive values of a vertically stretch ($a > 1$) or shrink ($a < 1$) the graph of f. h translates the graph of f left ($h < 0$) or right ($h > 0$), and k translates the graph of f up ($k > 0$) or down ($k < 0$). When a is negative, the graph of f is reflected in the x-axis.
2. The graph of $g(x) = \log_4(-x)$ is a reflection in the y-axis of the graph of $f(x) = \log_4 x$.

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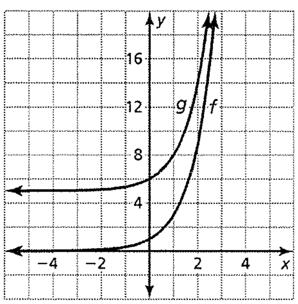
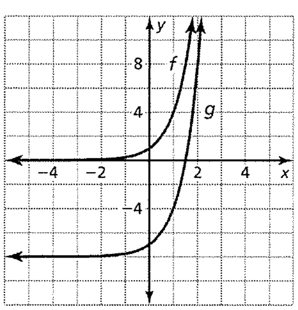
3. The graph is C because the parent function is translated left and down.
4. The graph is D because the parent function is translated left and up.
5. The graph is A because the parent function is translated right and down.
6. The graph is B because the parent function is translated right and up.
7. Notice that the function is of the form $g(x) = 3^x + k$. Because $k = 5$, the graph of g is a translation 5 units up of the graph of f.



8. Notice that the function is of the form $g(x) = 4^x + k$. Rewrite the function to identify k.

$$g(x) = 4^x + (-8)$$

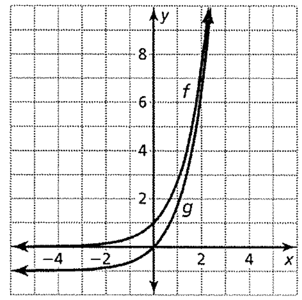
Because $k = -8$, the graph of g is a transformation 8 units down of the graph of f.



9. Notice that the function is of the form $g(x) = e^x + k$. Rewrite the function to identify k.

$$g(x) = e^x + (-1)$$

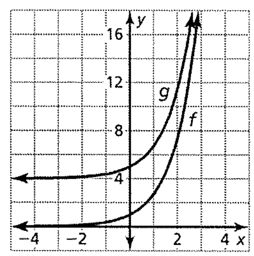
Because $k = -1$, the graph of g is a translation 1 unit down of the graph of f.



10. Notice that the function is of the form $g(x) = e^x + k$.

$$g(x) = e^x + 4$$

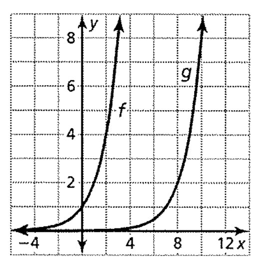
Because $k = 4$, the graph of g is a translation 4 units up of the graph of f.



11. Notice that the function is of the form $g(x) = 2^{x-h}$. Rewrite the function to identify h.

$$g(x) = 2^{x-7}$$

Because $h = 7$, the graph of g is a translation 7 units right of the graph of f.



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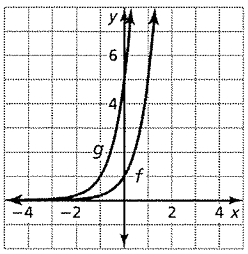
12. Notice that the function is of the form $g(x) = 5^{x-h}$. Rewrite the function to identify h.

$$g(x) = 5^{x-(-1)}$$

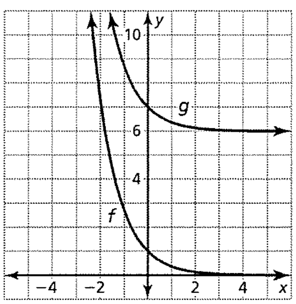
Because $h = -1$, the graph of g is a translation 1 unit left of the graph of f.



13. Notice that the function is of the form $g(x) = e^{-x} + k$.

$$g(x) = e^{-x} + 6$$

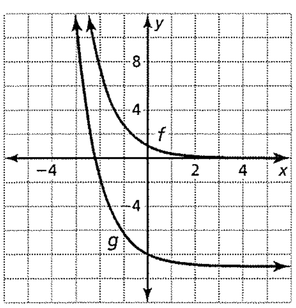
Because $k = 6$, the graph of g is a translation 6 units up of the graph of f.



14. Notice that the function is of the form $g(x) = e^{-x} + k$. Rewrite the function to identify k.

$$g(x) = e^{-x} + (-9)$$

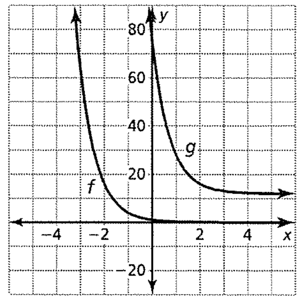
Because $k = -9$, the graph of g is a translation 9 units down of the graph of f.



15. Notice that the function is of the form $g(x) = \left(\frac{1}{4}\right)^{x-h} + k$.

$$g(x) = \left(\frac{1}{4}\right)^{x-3} + 12$$

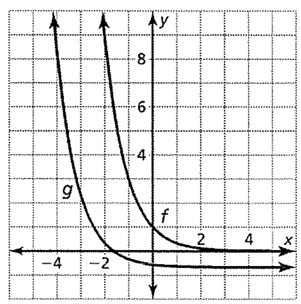
Because $h = 3$ and $k = 12$, the graph of g is a translation 3 units right and 12 units up of the graph of f.



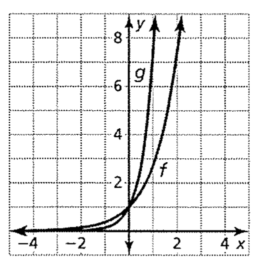
16. Notice that the function is of the form $g(x) = \left(\frac{1}{3}\right)^{x-h} + k$. Rewrite the function to identify h and k.

$$g(x) = \left(\frac{1}{3}\right)^{x-(-2)} + \left(-\frac{2}{3}\right)$$

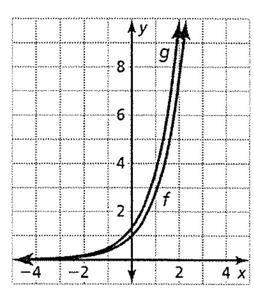
Because $h = -2$ and $k = -\frac{2}{3}$, the graph of g is a translation 2 units left and $\frac{2}{3}$ unit down of the graph of f.



17. Notice that the function is of the form $g(x) = e^{ax}$, where $a = 2$. So, the graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f.



18. Notice that the function is of the form $g(x) = ae^x$, where $a = \frac{4}{3}$. So, the graph of g is a vertical stretch by a factor of $\frac{4}{3}$ of the graph of f.



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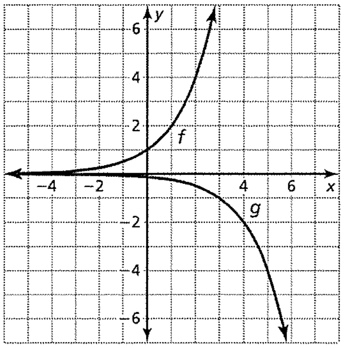
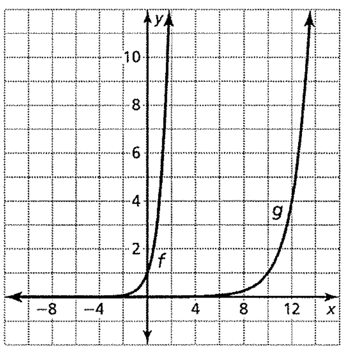
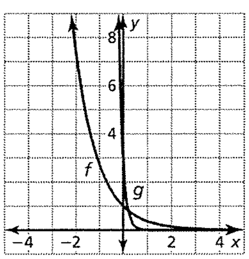
19. Notice that the function is of the form $g(x) = -2^{x-h}$, where $h = 3$. So, the graph of g is a reflection in the x-axis followed by a translation 3 units right of the graph of f.



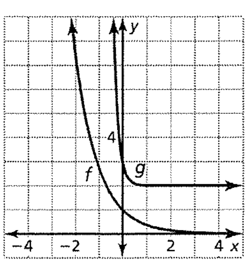
20. Notice that the function is of the form $g(x) = 4^{ax-h}$, where $a = 0.5$ and $h = 5$. So, the graph of g is a horizontal stretch by a factor of 2 followed by a translation 5 units right of the graph of f.



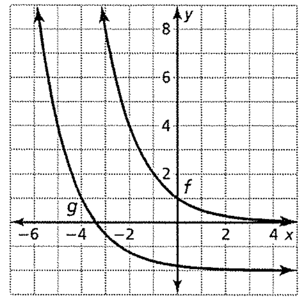
21. Notice that the function is of the form $g(x) = ae^{-bx}$, where $a = 3$ and $b = 6$. So, the graph of g is a horizontal shrink by a factor of $\frac{1}{6}$ and a vertical stretch by a factor of 3 of the graph of f.



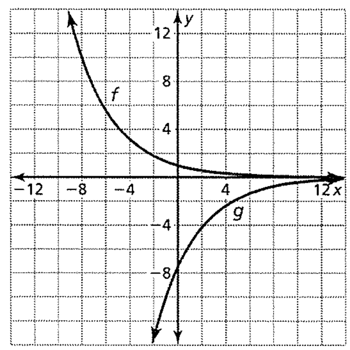
22. Notice that the function is of the form $g(x) = e^{-ax} + k$, where $a = 5$ and $k = 2$. So, the graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 2 units up of the graph of f.



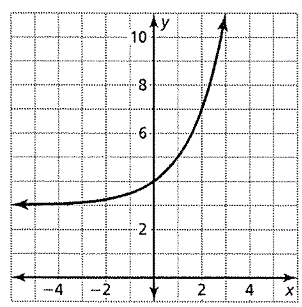
23. Notice that the function is of the form $g(x) = a\left(\frac{1}{2}\right)^{x-h} + k$, where $a = 6$, $h = -5$, and $k = -2$. So, the graph of g is a vertical stretch by a factor of 6 and a translation 5 units left and 2 units down of the graph of f.



24. Notice that the function is of the form $g(x) = -\left(\frac{3}{4}\right)^{x-h} + k$, where $h = 7$ and $k = 1$. So, the graph of g is a reflection in the x-axis followed by a translation 7 units right and 1 unit up of the graph of f.



25. The error is that the parent function was translated left instead of up.



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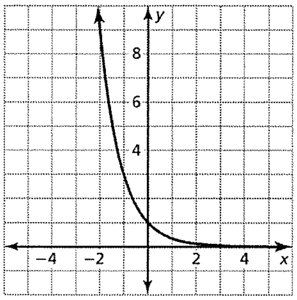
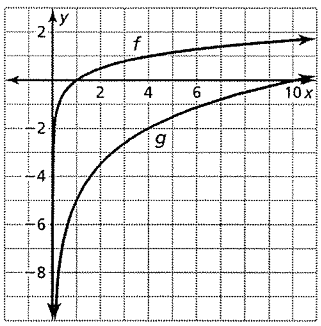
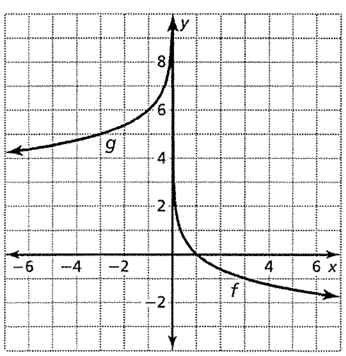
26. The error is in the reflection. The graph of the parent function $f(x) = 3^x$ should be reflected in the y-axis not the x-axis.



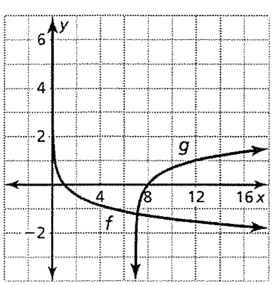
27. Notice that the function is of the form $g(x) = a \log_4 x + k$, where $a = 3$ and $k = -5$. So, the graph of g is a vertical stretch by a factor of 3 followed by a translation 5 units down of the graph of f.



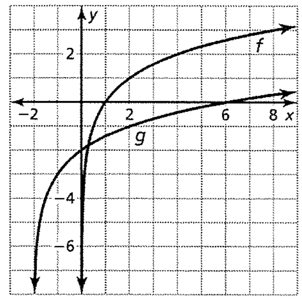
28. Notice that the function is of the form $g(x) = \log_{1/3}(-x) + k$, where $k = 6$. So, the graph of g is a reflection in the y-axis and a translation 6 units up of the graph of f.



29. Notice that the function is the form $g(x) = -\log_{1/5}(x - h)$, where $h = 7$. So, the graph of g is a reflection in the x-axis followed by a translation 7 units right of the graph of f.



30. Notice that the function is of the form $g(x) = \log_2(x - h) + k$, where $h = -2$ and $k = -3$. So, the graph of g is a translation 2 units left and 3 units down of the graph of f.



31. The graph is A because it is a translation to the right of the graph of f.

32. The graph is D because it is a translation to the left of the graph of f.

33. The graph is C because it is a vertical stretch of the graph of f.

34. The graph is B because it is a horizontal shrink of the graph of f.

35. Step 1 First write a function h that represents the translation of f.

$$h(x) = f(x) - 2$$
$$= 5^x - 2$$

Step 2 Then write a function g that represents the reflection of h.

$$g(x) = h(-x)$$
$$= 5^{-x} - 2$$

The transformed function is $g(x) = 5^{-x} - 2$.

36. Step 1 First write a function h that represents the reflection of f.

$$h(x) = -f(x)$$
$$= -\left(\frac{2}{3}\right)^x$$

Step 2 Then write a function g that represents the vertical stretch and translation of h.

$$g(x) = 6h(x + 4)$$
$$= -6\left(\frac{2}{3}\right)^{x+4}$$

The transformed function is $g(x) = -6\left(\frac{2}{3}\right)^{x+4}$.

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37. Step 1 First write a function h that represents the horizontal shrink of f.

$h(x) = f(2x) = e^{2x}$

Step 2 Then write a function g that represents the translation of h.

$g(x) = h(x) + 5$

$= e^{2x} + 5$

The transformed function is $g(x) = e^{2x} + 5$.

38. Step 1 First write a function h that represents the translation of f.

$h(x) = f(x - 4) - 1$

$= e^{-x+4} - 1$

Step 2 Then write a function g that represents the vertical shrink of h.

$g(x) = \frac{1}{3}h(x)$

$= \frac{1}{3}(e^{-x+4} - 1)$

$= \frac{1}{3}e^{-x+4} - \frac{1}{3}$

The transformed function is $g(x) = \frac{1}{3}e^{-x+4} - \frac{1}{3}$.

39. Step 1 First write a function h that represents a vertical stretch of f.

$h(x) = 6f(x)$

$= 6 \log_6 x$

Step 2 Then write a function g that represents a translation of h.

$g(x) = h(x) - 5$

$= 6 \log_6 x - 5$

The transformed function is $g(x) = 6 \log_6 x - 5$.

40. Step 1 First write a function h that represents a reflection of f.

$h(x) = -f(x)$

$= -\log_5 x$

Step 2 Then write a function g that represents a translation of g.

$g(x) = h(x + 9)$

$= -\log_5(x + 9)$

The transformed function is $g(x) = -\log_5(x + 9)$.

41. Step 1 First write a function h that represents a translation of f.

$h(x) = f(x + 3) + 2$

$= \log_{1/2}(x + 3) + 2$

Step 2 Then write a function g that represents a reflection of h.

$g(x) = h(-x)$

$= \log_{1/2}(-x + 3) + 2$

The transformed function is $g(x) = \log_{1/2}(-x + 3) + 2$.

42. Step 1 First write a function h that represents a translation of f.

$h(x) = f(x - 3) + 1$

$= \ln(x - 3) + 1$

Step 2 Then write a function g that represents a horizontal stretch of h.

$g(x) = h\left(\frac{1}{8}x\right)$

$= \ln\left(\frac{1}{8}x - 3\right) + 1$

The transformed function is $g(x) = \ln\left(\frac{1}{8}x - 3\right) + 1$.

43. $h(x) = -f(x)$ — Multiply the output by -1.

$= -\log_7 x$ — Substitute $\log_7 x$ for $f(x)$.

$g(x) = h(x) - 6$ — Subtract 6 from the output.

$= -\log_7 x - 6$ — Substitute $-\log_7 x$ for $h(x)$.

44. $h(x) = 4f(x)$ — Multiply the output by 4.

$= 4 \cdot 8^x$ — Substitute 8^x for $f(x)$.

$g(x) = h(x + 3) + 1$ — Add 3 to the input and 1 to the output.

$= 4 \cdot 8^{x+3} + 1$ — Replace x with $x + 3$ in $h(x)$ and add 1 to the output.

45. The graph of g is a translation 4 units up of the graph of f. The asymptote is $y = 4$.

46. The graph of g is a translation 9 units right of the graph of f. The asymptote is $y = 0$.

47. The graph of g is a translation 6 units left of the graph of f. The asymptote is $x = -6$.

48. The graph of g is a translation 13 units up of the graph of f. The asymptote is $x = 0$.

49. The transformation of f is a vertical shrink by a factor of 0.118 followed by a translation 0.159 unit up is S.

Sand particle	Diameter (mm), d	S
Fine sand	0.125	0.052
Medium sand	0.25	0.088
Coarse sand	0.5	0.123
Very coarse sand	1	0.159

50 a. The transformation is a reflection in the y-axis.

b. no; The result will still be a reflection in the y-axis.

51. Your friend is correct. *Sample answer:* A vertical translation will result in graphs that do not intersect.

52. yes; $\ln x$ and e^x are inverses, so they are reflections of each other in the line $y = x$.

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53. a. never; A vertical translation does not affect a vertical asymptote.

b. always; A vertical translation does affect a horizontal asymptote.

c. always; A horizontal shrink is obtained by multiplying x by a constant, which does not change the domain.

d. sometimes; Because there is a vertical translation, it is possible for the graph to have an x-intercept.

54. a. The domain is $t \geq 0$ and the range is $0 \leq P \leq 100$.

b. $P = 100(0.99997)^{12{,}000}$

≈ 69.77

There are about 69.77 grams left after 12,000 years.

c. The transformation would be a vertical stretch by a factor of $\frac{55}{100} = 5.5$.

d. The transformation does not affect the domain, but changes the range to $0 \leq P \leq 550$.

55. The function h is a translation 2 units left of f_1 and h is a reflection in the y-axis followed by a translation 2 units left of g. Rewrite h as $h(x) = f(x + 2)$ and as $h(x) = g[-(x + 2)]$.

56. *Sample answer:* $y = 3 \cdot 2^x + 2$

Maintaining Mathematical Proficiency

57. $(fg)(x) = f(x)g(x)$

$= x^4 \cdot x^2$

$= x^6$

When $x = 3$, the value of the product is

$(fg)(3) = 3^6 = 729$.

58. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{4x^6}{2x^3}$

$= 2x^3$

When $x = 5$, the value of the quotient is

$\left(\frac{f}{g}\right)(5) = 2(5)^3 = 250$.

59. $(f + g)(x) = f(x) + g(x)$

$= 6x^3 + 8x^3$

$= 14x^3$

When $x = 2$, the value of the sum is

$(f + g)(2) = 14(2)^3 = 112$.

60. $(f - g)(x) = f(x) - g(x)$

$= 2x^2 - 3x^2$

$= -x^2$

When $x = 6$, the value of the difference is

$(f - g)(6) = -6^2 = -36$.

6.1–6.4 What Did You Learn? (p. 325)

1. Calculating 10% of 233 million gives 23.3 million. Because the number of cell phone users was increasing by only 6%, the number of users should not increase by that many in the several years after 2006.

2. Because the exponents in the equations in Exercises 23 and 26 are positive, the graphs must show growth. Because the exponents in Exercises 24 and 25 are negative, the graphs must show decay. Substituting 0 for x to find the y-intercept can also be used to match the graphs.

3. Find the coordinates when $x = 1$ and $x = 10$. Then find the rate of change. Compare the rates of each function.

6.1–6.4 Quiz (p. 326)

1. The function represents exponential growth because the base, 4.25, is greater than 1.

2. The function represents exponential decay because the base, $\frac{3}{8}$, is greater than 0 and less than 1.

3. The function represents exponential growth because the base, $e^{0.6}$, is greater than 1.

4. The function represents exponential decay because the base, e^{-2}, is greater than 0 and less than 1.

5. $e^8 \cdot e^4 = e^{8+4} = e^{12}$

6. $\frac{15e^3}{3e} = 5e^{3-1} = 5e^2$

7. $(5e^{4x})^3 = 5^3e^{3(4x)} = 125e^{12x}$

8. $e^{\ln 9} = 9$

9. $\log_7 49^x = \log_7(7^2)^x = \log_7 7^{2x} = 2x$

10. $\log_3 81^{-2x} = \log_3(3^4)^{-2x} = \log_3 3^{-8x} = -8x$

11. $\log_4 1024 = 5$, so $4^5 = 1024$.

12. $\log_{1/3} 27 = -3$, so $\left(\frac{1}{3}\right)^{-3} = 27$.

13. $7^4 = 2401$, so $\log_7 2401 = 4$.

14. $4^{-2} = 0.0625$, so $\log_4 0.0625 = -2$.

15. $\log 45 \approx 1.653$

16. $\ln 1.4 \approx 0.336$

17. $\log_2 32 = \log_2 2^5 = 5$